Today classical EU
Rational!

07 Nov 12 1

Explain back book
+ try yourself

R & R 2

9:30-
12:00

Homework: Let a student do

Ex. 1.7.3 e; p.16 $x \succsim y,\ y \succ z \Rightarrow x \succ z$

1.6.1 (p. 28: DB > maxmin)

Ass. 1.6.8: CE additive
p. 30

implies $\succsim$ add:

$x \succsim y \Leftrightarrow CE(x) \geq CE(y) \Leftrightarrow$
$CE(x) + CE(z) \geq CE(y) + CE(z) \Leftrightarrow$
$CE(x+z) \geq CE(y+z) \Leftrightarrow$
$x+z \succsim y+z$

$\succsim$ add implies CE additive

$x \sim CE(x)$
$y \sim CE(y)$

Ex. 1.5.1

$x+y \sim CE(x) + CE(y)$
This implies
$CE(x+y) = CE(x) + CE(y)$

Easier: $\succsim$ add implies EV (t×m) ⇒ CE is add.

2012 9:46 ± 9:30 2011

Ch. 2. Today classical EU Rational!
Many of you had before. But presented differently: observability! Empirical status
Important def: large stakes.
Additivity not reasonable. No EV
nonquantitative: No EV.

generalization
For simplicity: prob^s known:
DUR

Ch 1: read minds.
Ch 4: read hearts
People in the literature do not really understand.
I explain in some detail because important later. (ambiguity: no ^{s} prob^s is a way more fundamental than discussed so far)

Vendor with prob given
$p_1 = \frac{1}{4}$ $p_2 = \frac{1}{2}$ $p_3 = \frac{1}{4}$
They take Table 1.1.1 (p.11) before them.
x: $(\frac{1}{4}: 400, \frac{1}{2}: 100, \frac{1}{4}: -400)$
y: $(\frac{1}{4}: -400, \frac{1}{2}: 100, \frac{1}{4}: 400)$
$x+y$: $(\frac{1}{4}: 0, \frac{1}{2}: 200, \frac{1}{4}: 0)$

Write only them. prob-contingent prospects.
DUR: only generated prob. distr. matters. $x = y$

$(p_1 : x_1, \dots, p_n : x_n) = p_1 x_1 \dots p_n x_n =$

$(p : \alpha, 1-p : \beta) = \alpha_p \beta$

$1 : \alpha = \xrightarrow{1} \alpha$: degenerate $\tilde{\alpha}$

So you saw an example of how we get prob distr^s over outcomes from ~~DUR ⊂ DUU!~~ states & DUU.

DUR: people only take prob dist^s over outcomes.
Pref^s over those.
Don't mention states.

I claim: DUR ⊂ DUU

skip this and p.4. Too abstract
Better do here Ex. 1.

To convince you: Every set of prob distr^s over outcomes is generated by S, acts (Str. Ass. 1.2.1)?

Yes we can!
E.g. $S = [0,1]$, P is uniform. $F^{-1} \Rightarrow$ every prob distr. over $\mathbb{R}$.

DUR CONCL: mathematically correct. conceptually useful. (What is prob? amb. ...!) 4

Subj. prob = obj. prob. if exists.

Again: people in the literature do not really understand

$E_1, \ldots, E_6$ of die. Obj. prob. $p_j = 1/6$.

Subj. prob. Q. Does DUR + SEV

$1_{E_i} 0 \sim 1_{E_j} 0 \Rightarrow Q(E_i) = Q(E_j)$

I do not really use SEV. Only stoch. dom.

$\Rightarrow Q(E_j) = 1/6$

~ for any n iso 6.

$E_1, \ldots, E_n$ $p_j = \frac{1}{n}$

$P(E_1 \cup \cdots \cup E_m) \doteq Q(E_1 \cup \cdots \cup E_m) = \frac{m}{n}$

$Q \doteq P$ for all rationals

limit: $Q \doteq P$. real nrs are uniquely sandwiched between rationals

Whenever enough richness!

Subj. prob is not different than obj. prob. Can't just do whatever you like. It is an extension obj. prob ⊂ subj. prob.

10:20

10:17

10:28

You are my witnesses:
I did not yet say what EU is!

1. aU: $EU(x) \geq x$

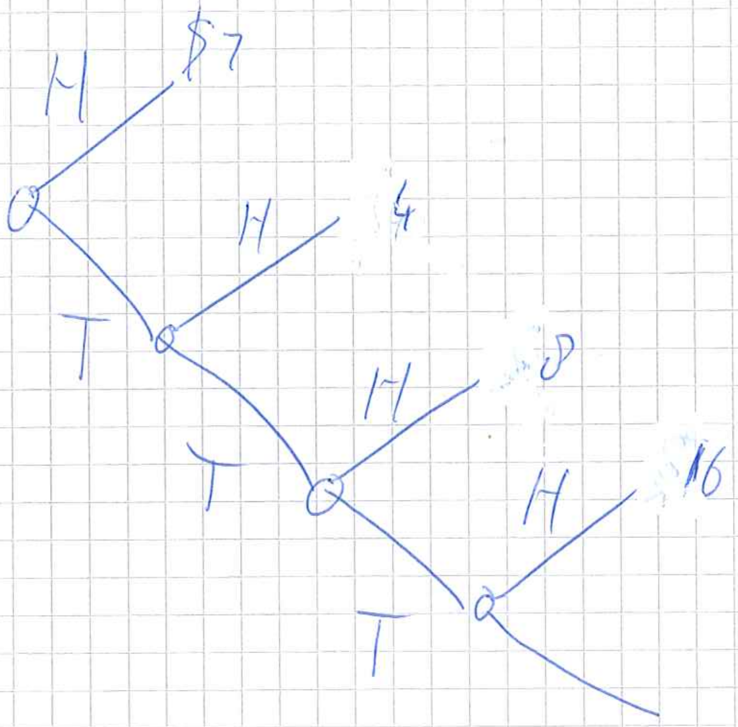
2. neutr: $EU(x) \sim x$

3. sk: $EU(x) \leq x$

I gave you def. in $\geq$,
not in theoretical terms.
No EU!

5.

St. Petersburg paradox



What is your CE?

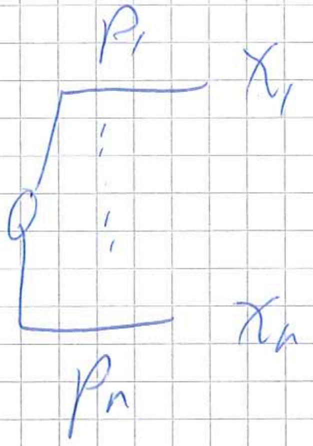
± $4 . . . .

$EV = \frac{1}{2} \times 2 + \frac{1}{4} \times 4 + \frac{1}{8} \times 8 + \dots + 2^{-j} 2^{j} + \dots =$

$1 + 1 + 1 + \dots + 1 + \dots = \infty$

$\neq$ EV!

EU_i



$\longrightarrow p_1 U(x_1) + \cdots + p_n U(x_n)$

$U = \ln$.

St. petersburg paradox.

$\sum 2^{-i} \ln 2^i = \sim = \ln 4$

So 4 is CE!

Explains the paradox.

Assume EU.

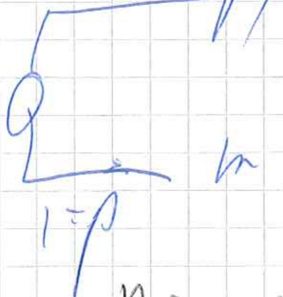
$U(0) = 0$; $U(100) = 1$.

How elicit $U(30)$?

Experimental heaven as in Ch. 1.

SG method:

$U(m) = 0$; $U(M) = 1$

$\alpha \sim$ 

→ Keep this on white-board

P&U, outcome & prob. become commensurable.

McCord & de Neufville (1986):

We are not in exp. heaven.

Noise!

Especially in

Say algebra, then $U(\alpha) = p$, in words

SG consistency:

replace "better" by "$\Rightarrow$"

Normative appeal!

vNM surprise

Thm: Two statements are equivalent:

(i) EU holds

(ii) $\succsim$ satisfies:

- weak ordering
- SG solvability
- SG dominance
- SG consistency

My course more Equitable than micro I. Observability!
* too bombastic!? empirical status!

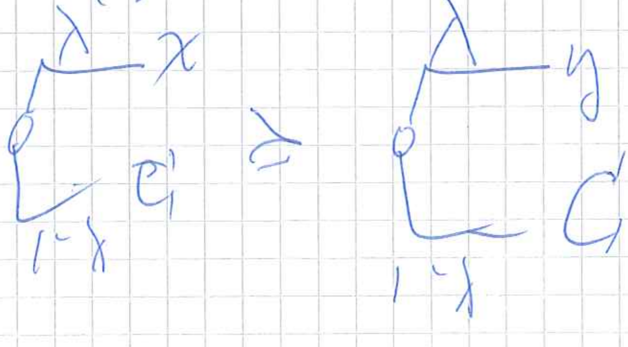
U cardinal: Can do or leave

11:00 break

Implication: → better done before allais paradox

Independence:

$x \succ y \Rightarrow$



10:55

11.

Healtl application

(§ 3.1. Powerpoint slides)

11.i.24

Thm. $\exists\, U$:

(i) U concave

(ii) r. a.v.

U lin.: r. neutr

U conv.: r. sk

Do everything about risk aversion in 5 minutes

$\alpha \sim_1 x \Rightarrow \alpha \succsim_2 x$

$\succsim_2$ MRA than $\succsim_1$ $\quad$ $\succsim_2$ has lower CE's

Thm. EU: $\succsim_2$ MRA $\succsim_1$ $\Longleftrightarrow$

$$U_2 = \varphi \circ U_1 \quad \text{with } \varphi \text{ concave}$$

$$\Longleftrightarrow$$

$$\frac{-U_2''}{U_2'} \geq \frac{-U_1''}{U_1'}$$

(Pratt-Arrow) index of abs. r.av. — better say: of concavity

(de Finetti 1952)

$$\Longleftrightarrow$$

(on $\mathbb{R}^+$): $$\frac{-U_2''(\alpha)}{U_2'(\alpha)} \times \alpha \geq \frac{-U_1''(\alpha)}{U_1'(\alpha)} \times \alpha$$

Index of relative proportional r.av.

$\Leftrightarrow$ $\succsim_2$ has bigger risk premium than $\succsim_1$

$EV(x) - CE(x) =$ risk premium

(of course: lower CE's of x $\Leftrightarrow$ higher risk premiums)

Check in book.

Decreasing abs. r. av:

$\left[x \sim \alpha \Rightarrow x + \varepsilon \succsim \alpha + \varepsilon \;\; \forall \varepsilon \geq 0\right]$

$\Leftrightarrow$

$-\frac{u''}{u'}$ is decreasing

Incr. rel. r. av:

$\left[x \sim \alpha \Rightarrow \lambda x \precsim \lambda \alpha \;\; \forall x \geq 0, \alpha \geq 0, \lambda \geq 1\right]$

$\Leftrightarrow -\frac{u''}{u'}\alpha$ is decr.

± 25 min
voor § 4.1