Notes for 1^{st} meeting of Prospect Theory: for Risk and Ambiguity

1^{st} meeting

The first meeting starts with de Finetti's famous book making (Ch. 1) that has impressed generations of researchers, and is well suited to impress newcomers. I teach this meeting along the lines explained in §1.8, p. 35 2^{nd} para. It works very well to attract students, and the second meeting I usually have more students than the first.

GENERAL TOPIC OF WHOLE COURSE: DUR/DUU individual; behavioral-revolution style

Only individual (may be organisation); no games/strategies; no intertemporal, no welfare. Many general things that you learn here for risk (rational/irrational; paternalism; behavioral style) pertain to the other fields (intertemporal, welfare, and so on) just as much.

PURPOSES:

- Prescriptive:

Do DUR/U well yourself; or as consultant (oil drilling); or to make patients follow prescriptions; and so on.

- Descriptive:

Describe-predict others (predict consumers' choices in marketing)

- Conceptual. Give empirical content of U & P.

Another claim I make with this course is that you understand the subtle P, and also U, better; U grew over centuries; marginal revolution; ordinal revolution; heroine ≻ medicine???; property of object or of DM? If you like history of economics. This course relates every theoretical concept directly to the empirical primitive. (2011 a student asked for literature.)

- SEU classical homo economicus (refer to their preceding micro-econ courses, if ...)
- PT: 1^{st} rational theory of irrational behavior; homo sapiens replaces homo economicus. This course will teach you. Hope you can then use it in whatever field you are working.

Optional: Only since '92, new dimension: chance attitude. Sorely missing. Mostly due to Schmeidler. Unlike Nash eq., has to *replace* things existing. Everyone in this room knows more about economics than I do. You will apply in your field.

Nice feature of decision theory: YOU. You're always involved; always: what would you do?

You will participate in some experiments. We will analyze your risk attitude.

Optional:
Prerequisites:
- basic probs (or statistics)
- basic algebra
- exact reasoning

I explain levels a, b, c of p. 4, allowing students to choose their level to take this course. (a-students have easier exercises but have to analyze data sets and have to know empirical findings better, than b, and they more than c).
Tell them to do homework exercises by themselves without checking solutions at the end. If they don't know how to solve, still don't check out solutions. Instead try to discover which part of the theory has been misunderstood. From this stage of searching in theory for what one misses, one learns most.

Above general intro takes 15 minutes.

Show most of following tables by computer, and not written. Vendor is toy example, but finance example is needed to show that it is serious.

EXAMPLE 1.1.1 [Vendor].

TABLE 1.1.1 (Blackboard). Net profits obtained from merchandise, depending on the weather

	no rain (s_1)	*some rain* (s_2)	*all rain* (s_3)
x (“ice cream”)	400	100	–400
y (“hot dogs”)	–400	100	400
0 (“neither”)	0	0	0
x + y (“both”)	0	200	0

We assume no demand-interactions

EXAMPLE 1.1.2 [Finance] (do on computer screen). Speculate on copper price next month.

TABLE 1.1.2. Net profits depending on copper price

	price $\geq$ *2.53* (E_1)	*2.53 > price* $\geq$ *2.47* (E_2)	*2.47 > price* (E_3)
x	50K	–30K	–30K
y	–30K	–30K	50K
0 (“neither”)	0	0	0
x + y (“both”)	20K	–60K	20K

□

Finance: Payment need not be continuous. Can have it this discrete.
Note that later in the finance-interpretation the market-price is NOT included in the prospect; it is what market has to decide on. In beginning leave open but suggest otherwise.

Discuss *briefly*: What would you do? Press them into more likely, and then harass them on claiming on probabilities that however they do not know.

Define EV for vendor.

Can do bad advise here, prior to elicitation, if danger of losing audience.
Otherwise better after elicitation.

Especially in the beginning, when first presenting decision models, I follow the 5-step procedure (see p. 7 para 4 ff.). Many teachers make the didactical mistake of thinking that step 2 (derive decision from model) is just calculation and is too trivial. Students really have to manually themselves derive a decision from a decision model before they understand how this works. Students themselves are not aware of this and find it trivial too, but still one has to do this when teaching.

p_j: subjective parameters.

EXERCISE 1.3.1.[a] Vendor Example: EV;
p_1 = P(no rain) = 0.40
p_2 = P(some rain) = 0.30
p_3 = P(all rain) = 0.30.

Calculate EV. What is chosen?

ELABORATION:

$EV(x) = 0.40 \times 400 + 0.30 \times 100 + 0.30 \times (-400) = 70;$

$EV(y) = 0.40 \times (-400) + 0.30 \times 100 + 0.30 \times 400 = -10.$

$EV(0) = 0;$

$EV(x+y) = 0.30 \times 200 = 60.$

x is chosen.

Probably skip the following.

Either following one with student nr. or from book, $p_1 = ½$, $p_2 = ¼$, $p_3 = ¼$.

EXERCISE 1.3.2.[a] Vendor Example. Assume EV with

$p_1 = \#/100$ with # last two digits of your student nr. If your student nr is 323262pw

then $p_1 = 0.62$.

$p_2 = 0$.

x = (400,0,0)

y = (0,0,400)

Calculate EV's.

Indicate what is chosen.

□

Roughly here usually ended 1st hour; can be some later.

NOW WE REVERSE.

Elicitation: explain that as if. Directly asking for subjective probs. does not work then. (Come to this some later telling that every noncon dept. would immediately have said this.)

Want to find out some about p_1, p_2, and p_3 of street vendor. You know he does EV.

The *elicitation-method* (you now learn mind-reading!)

EXERCISE 1.3.3. x = (400,0,0) $\succ$ (0,0,400) = y. Show EV $\Rightarrow p_1 > p_3$. □

2011 TI: I forgot to say the mind reading.

$x \succ y \;\; \Rightarrow \;\; p_1 > p_3$ (explain preference symbol).

Here 1st hour also ends sometimes.

Want to find out p_1, p_2, and p_3 of street vendor exactly. You know he does EV.

You are in experimental heaven (book p. 36 3rd para).:

Can observe all prefs between all prospects. All CEs. Explain this well!

How proceed? Let them guess. Often someone points out that getting three indifferences leads to three equalities that may be hoped to be linearly independent. Then someone notes that, because of unit sum, two equalities suffice.

Find

$p \sim (1,0,0)$. similar to (1.3.1)

Then

$p = P(s_1) \times 1 + (1 - P(s_1)) \times 0 = P(s_1)$. similar to (1.3.2)

p: willingness to bet

Can skip following. Depends on time.

EXERCISE 1.3.5.[a]

$(100,0,0) \sim 50$

$(0,100,0) \sim 25$.

What are p_1, p_2, p_3?

CE(0,0,100)?

Preference between (0,100,0) and (0,0,100)?

$p_1 100 + (1-p_1)0 = 50 \Rightarrow p_1 = ½$.

$p_2 100 + (1-p_2)0 = 25 \Rightarrow p_2 = ¼$.

$p_3 = 1 - ½ - ¼ = ¼$.

$CE(0,0,100) = ¼ \times 100 = 25$.

$(0,100,0) \sim (0,0,100)$.

□

EXERCISE. # is last two digits of your student nr.

$(100,0,0) \sim \#$.

$(0,100,0) \sim 0$.

What are p_1, p_2, p_3?

CE(0,0,100)?

Preference between (100,0,0) and (0,0,100)?

□

Subjective probability measurements: weather forecasters, oil drilling, government.
Can talk some longer, such as calibration.

Sometimes only here 1st hour ends.

Advisor

Tell them they hire a decision analyst who advises them.

Advisor: Maximize EV!
Are you convinced?

Discuss.

No! Ad hoc!
p's meaningless. Why operations as are?

Law of large nrs?
- Repetitions;
- weather ≠ repeatable
- sumtotal linear utility? Starve first 2 years, millionaire in 3rd: 3rd yr doesn't help.

Need different story.

Here often 2nd hour ended.

Advisor apologizes. Change subject! He now gives a very different, and much better, more meaningful, advise.

Bookmaking / no-Arbitrage

Normative.

1. Monotonicity: OK

2. Transitivity: OK.

Preparation for next:

2. Addition of prospects.

Already seen in x+y in vendor & finance.

Additivity:

$[x \succcurlyeq y \;\Rightarrow\; x+z \succcurlyeq y+z]$ for all prospects x,y,z. □

Improving ingredient in sum improves the sum.

TABLE 1.5.1.

		no rain	*some rain*	*all rain*			*no rain*	*some rain*	*all rain*
If	x	400	100	–400	$\succcurlyeq$	y	–400	100	400
	z	500	500	500		z	500	500	500
then	x+z	900	600	100	$\succcurlyeq$	y+z	100	600	900

x: ice cream y: hot dogs

z: message from the tax authorities that you receive a tax credit of $500.

In both cases, llh. (whatever that may mean …) of no rain vs. all rain decides. Seems plausible. Would yo rather have the best prize under *no-rain* or under *all-rain*?

TABLE 1.5.2.

		E_1	E_2	E_3			E_1	E_2	E_3
If	x	50K	–30K	–30K	$\succcurlyeq$	y	–30K	–30K	50K
	z	30K	30K	30K		z	30K	30K	30K
then	x+z	80K	0	0	$\succcurlyeq$	y+z	0	0	80K

z: message from the tax authorities that you receive a tax credit of 30K.

In both cases, llh. (whatever that may mean …) of E_1 vs. E_3 decides. Seems plausible.

z was constant. Need not be.

TABLE 1.5.3.

		no rain	*some rain*	*all rain*			*no rain*	*some rain*	*all rain*
If	x	400	100	–400	$\succcurlyeq$	y	–400	100	400
	z	150	100	50		z	150	100	50
then	x+z	550	200	–350	$\succcurlyeq$	y+z	–250	200	450

x: ice cream y: hot dogs

z: magazines

More **questionable**, and not 100% convincing to all of you. x+z is more risky than y+z. This point is more questionable, but also more interesting! Here additivity and risk-perception clash. Here there is something to really learn.

Don't ask them to be convinced, but:

Please give me this one for now.

Reasonable for moderate amounts

Now counterexample (either following flood example that has disappeared from book or, better, the finance example following).

skip flood, do finance

TABLE removed from book

		no rain	*some rain*	*all rain*			*no rain*	*some rain*	*all rain*
	x	400	100	–400	≽	y	–400	100	400
	z	0	0	–10,000		z	0	0	–10,000
but	x + z	400	100	–10,400	≺	y + z	–400	100	–9,600

x: ice cream y: hot dogs

z: The water is very high. If there is all rain tomorrow, your boot will be flooded, costing $10,000.

z leverage to left, hedge to right

TABLE 1.5.4.

		E_1	E_2	E_3			E_1	E_2	E_3
If	x	50K	–30K	–30K	≽	y	–30K	–30K	50K
	z	40K	0	–40K		z	40K	0	–40K
then?	x + z	90K	–30K	–70K	≽?	y + z	10K	–30K	10K

z leverage for x

z hedge for y.

Assume < –40K: bankrupt!

Then x + z ≺ y + z.

Additivity fails.

Nice: say that in Table 1.5.3 it was questionable, and Table 1.5.4 it undoubtedly was **wrong**. Later tell them to be convinced in Table 1.5.3 but not 1.5.4. This build-up line is very nice.

OK, we assume moderate amounts henceforth.

They will not understand for a while why all that algebra (to ome) is being done. Remind them every now and then that we are exploring implications, to discover something remarkable.

Here may be end of 2nd hour. Can also come later.

Explain superscripts.

The following is too abstract for the class. So do proof with first $x^1+x^2 \succcurlyeq y^1+y^2$, next $x^1+x^2+x^3 \succcurlyeq y^1+y^2+y^3$, and so on. This is a case where presentation in class is to be different than writing in a book.

EXERCISE 1.5.1.[!b] Assume that $\succcurlyeq$ is transitive and additive. Prove:

a) $\mathbf{x^1} \succcurlyeq \mathbf{y^1}$ then $\mathbf{x^1} + y^2 + \cdots + y^n \succcurlyeq \mathbf{y^1} + y^2 + \cdots + y^n$.

b) [Improving Several Prospects in a Sum of Prospects Improves the Whole Sum]. If $x^i \succcurlyeq y^i$ for all i then $x^1 + \cdots + x^m \succcurlyeq y^1 + \cdots + y^m$. Table 1.5.5 illustrates this result for $m = 2$. □

Skip

TABLE 1.5.5.

		no rain	*some rain*	*all rain*			*no rain*	*some rain*	*all rain*
If	x^1	400	100	–400	$\succcurlyeq$	y^1	–400	100	400
and	x^2	200	100	0	$\succcurlyeq$	y^2	50	100	150
then	$x^1 + x^2$	600	200	–400	$\succcurlyeq$	$y^1 + y^2$	–350	200	550

x^1: ice cream y^1: hot dogs

x^2: newspapers y^2: umbrellas

Now a particular way of violation of the above.

Imagine you discover in your well-contemplated prefs :

$$x^1 \succcurlyeq y^1$$
$$x^2 \succcurlyeq y^2$$
$$\vdots$$
$$x^m \succcurlyeq y^m$$

$\sum_{j=1}^{m} x^j(s) < \sum_{j=1}^{m} y^j(s)$ for all $s \in S$ (can be > 3 events).

Write the inequalities for all three events:

$$\begin{array}{cccccc}
s_1 & s_2 & s_3 & s_1 & s_2 & s_3 \\
x^1{}_1 & x^1{}_2 & x^1{}_3 & y^1{}_1 & y^1{}_2 & y^1{}_3 \\
+ & + & + & + & + & + \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
+ & + & + & + & + & + \\
x^n{}_1 & x^n{}_2 & x^n{}_3 & y^n{}_1 & y^n{}_2 & y^n{}_3 \\
\overline{}\ + & \overline{}\ + & \overline{}\ + & \overline{}\ + & \overline{}\ + & \overline{}\ +
\end{array}$$

The above vertical notation does not come naturally. More natural, first writing x'es, then y's, and only then inequalities.

s_1 $\quad x^1{}_1 + \cdots + x^n{}_1 \quad < \quad y^1{}_1 + \cdots + y^n{}_1$

s_2: $\quad x^1{}_2 + \ldots + x^n{}_2 \quad < \quad y^1{}_2 + \ldots + y^n{}_2$

s_3: $\quad x^1{}_3 + \ldots + x^n{}_3 \quad < \quad y^1{}_3 + \ldots + y^n{}_3$

Lo and behold!

Arbitrage/Dutch book

TABLE 1.5.4.

E_1	E_2	E_3		E_1	E_2	E_3
0	0	0	$\succcurlyeq$	–100	300	300
10	20	20		200	–100	200
20	20	10	$\succcurlyeq$?	200	200	–100
			+			
30	40	30		300	400	400

Say line that DB is just an implication, telling them the line.

FIGURE 1.5.1. Arbitrage (a Dutch book)

$$\begin{array}{ccc}
(x_1^1, \ldots, x_n^1) & \succcurlyeq & (y_1^1, \ldots, y_n^1) \\
 & \cdot & \\
 & \cdot & \\
 & \cdot & \\
(x_1^m, \ldots, x_n^m) & \succcurlyeq & (y_1^m, \ldots, y_n^m) \\
\text{—— +} \quad \text{—— +} & & \text{—— +} \quad \text{—— +} \\
\Sigma x_1^j \;\ldots\; \Sigma x_n^j & & \Sigma y_1^j \;\ldots\; \Sigma y_n^j
\end{array}$$

Here can be end of 2nd hour.

Can discuss bike and insurance in detail, but I prefer doing that later.

Can discuss arbitrage in financial market here, but I prefer doing it later

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1.6 de Finetti's Surprise

THEOREM 1.6.1 [De Finetti; No-Arbitrage]. Under Structural Assumption 1.2.1 (decision under uncertainty), the following three statements are equivalent.

(i) Expected value holds.

(ii) The binary relation $\succcurlyeq$ is a weak order, for each prospect there exists a certainty equivalent, and no arbitrage (Dutch book) is possible.

(iii) The binary relation $\succcurlyeq$ is a weak order, for each prospect there exists a certainty equivalent, and additivity and monotonicity are satisfied. □

Tell them I explained implication (iii) $\Rightarrow$ (ii), but not other implications. (A student said to be amazed that Book excludes EV, but then I explain through sums of EVs.)

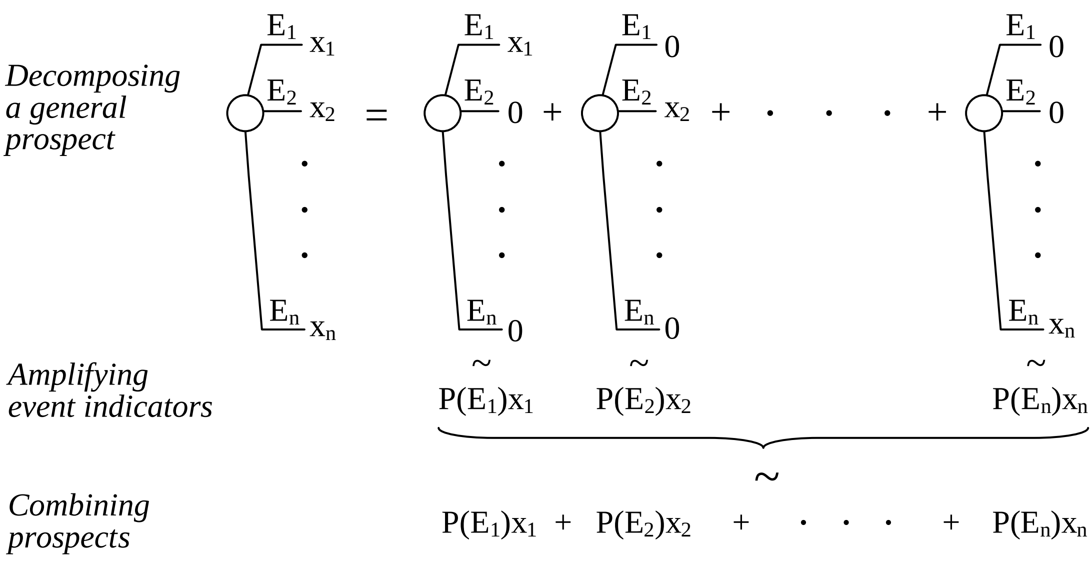
For following, have to explain what it means to sell short (sell skin of bear before having shot the bear).
EXERCISE. Imagine arbitrage in financial market. How can you make money? □

Discuss bike insurance.

Here I usually end. No time for following material.

EXERCISE 1.6.5^c
s_1: no rain tomorrow.
Your p_1 = #/100; or say general that EV with p_1.
You will receive the prospect $(1-(1-r)^2)_{s_1}(1-r^2)$. Which number r do you choose (to be expressed in terms of p_1).
□

FIGURE 1.11.1. Deriving expected value



Pref. foundation reclame: You may have seen models in your life, maybe some of you saw PT already. You may have shrugged your shoulders, without really being able to relate to whether the models are good or bad. We do everything through preference foundations, which give you a good feel about what the models mean.

If can, stress Structural Assumption 1.2.1 (p. 17).

I teach the above material in one session of 2.5 hours. The material is not very much, but best to leave it here. Better not to start with decision under risk (Ch. 1) so as not to confuse them conceptually.

Can already here let them do §4.1 as homework, time permitting.