Solutions to Extra Exercises for

Wakker (2010) "Prospect Theory: for Risk and Ambiguity"

August, 2023

EXERCISE 1.6.14. The combination of the two prospects gives 6 for sure. Because CE is additive, 6 must be the sum of the CEs. So, the CE of each is 3. □

EXERCISE 1.6.15. What John does is the principle of trade. It is no arbitrage because he did not combine preferences of one person, but of two different persons. If one could consider the pair {Peter, Paul} as one decision unit and they could easily have traded with each other, then it could have been argued that John had arbitraged this decision unit. But such is not the case. □

EXERCISE 2.3.3.

a) (1/3:100, 2/3:0) in all three cases.

b) The three state-contingent prospects all induce the same probability distribution over outcome and, by Assumption 2.1.2, are equivalent. Note that you cannot claim that indifference follows from identical expected values at this stage. The prospects have identical *objective* expected values using the objective p_j´s, but no-one said that such expected values are maximized by preferences. Preferences maximize subjective expected values based on the p_js.

c) The prospects in part b) have the same SEV. Hence, $p_1 100 = p_2 100 = p_3 100$, so that $p_1 = p_2 = p_3$. These probabilities must all be 1/3.

d) They are the same.

e) The prospects $100_{s_j}0$ are all indifferent, implying that all values $p_j 100$ are the same. Hence, all p_j are 1/n. Subjective and objective probabilities are identical.

f) Assumption 2.1.2, on decision under risk, alone already implies the equivalences under part b. It is natural to speculate that under most decision models using subjective probabilities, the three events must then have the same subjective probabilities also.—This holds under general models that satisfy Machina & Schmeidler's (1992) probabilistic sophistication, which assumes a sort of subjective stochastic dominance condition that is very plausible.—Then the rest of the exercise follows.

FURTHER COMMENT. If objective probabilities are available, then subjective probabilities usually have to agree with objective ones. For instance, if we have sufficient richness to have a uniform partition $\{E_1,\ldots,E_n\}$ with $P(E_j) = 1/n$ for all j, then all events with objective probability j/n have the same subjective probability j/n (being the same as of $E_1 \cup \cdots \cup E_j$). By monotonicity, the difference between objective and subjective probabilities then can never exceed 1/n.

This exercise provides an alternative way to show what Example 2.3.2, Exercise 2.3.1, and the para following it also show. □

EXERCISE 2.3.4.

a) There is risk aversion. It is strict (so, no risk neutrality) in the sense that, as soon as $x_1 \neq x_2$, i.e., $x_1 > x_2$, then the certainty equivalent of (x_1,x_2), being $0.4x_1 + 0.6x_2$, is strictly below the expected value $0.5x_1 + 0.5x_2$.

b) No arbitrage is possible. Arbitrage here would also be arbitrage against the model when extended to the whole set $\mathbb{R}^2$, i.e. if the domain included also prospects (x_1,x_2) with $x_1 < x_2$. Then Theorem 1.6.1 would be violated. A direct way to see this point is that if $SEV(x^j) \geq SEV(y^j)$ for all j, then also $SEV(x^1) + \cdots + SEV(x^n)$ $= SEV(x^1 + \cdots + x^n) \geq SEV(y^1 + \cdots + y^n) = SEV(y^1) + \cdots + SEV(y^n)$, but the latter inequality cannot be if $(y^1 + \cdots + xy^n) >> (x^1 + \cdots + x^n)$.

c) The decision under risk assumption 2.1.2 holds. If two event-contingent prospects induce the same probability distribution over outcomes, then that must be of the form $(0.5{:}x_1, 0.5{:}x_2)$ with $x_1 \geq x_2$, and then the "two" event-contingent prospect must be identical, both being (x_1,x_2). Hence, they are equivalent.

d) The difference is that in the Extra Exercise 2.3.3.e all elements of $\mathbb{R}^2$ occur as prospects, whereas here only a strict subset is considered. This is the only difference. When in Exercise 1.6.6 we constructed a Dutch book from risk aversion, we essentially used an act (β,γ) with $\gamma > \beta$, which served as a hedge, but such an act is not available in the domain considered here, and we have no hedges here.

COMMENT: Situations as in this exercise occur in finance, where only a set of really available event-contingent prospects is considered, which can be as above. Then the market can, for instance, choose the as-if-risk-neutral subjective probabilities more pessimistically than the objective ones so as to be strictly risk averse but yet without running into arbitrage. □

EXERCISE 2.6.7.

a. The indifference implies $U(30) = 0.40U(100) + 0.60U(0) = 0.40$.

b. The indifference implies (immediately crossing out a common term)

$$\tfrac{1}{2} \times U(30) + \cancel{\tfrac{1}{2} \times U(0)} = \tfrac{1}{2} \times 0.40 \times U(100) + \tfrac{1}{2} \times 0.60 \times U(0) + \cancel{\tfrac{1}{2} \times U(0)},$$

so,

$$\tfrac{1}{2} \times U(30) = \tfrac{1}{2} \times \big(0.40 \times U(100)\big) = \tfrac{1}{2} \times 0.40$$

so,

$$U(30) = 0.40.$$

c. The indifference implies $U(70) = 0.80U(100) + 0.20U(0) = 0.80$.

d. The indifference implies (immediately crossing out a common term)

$$\tfrac{1}{2} \times U(70) + \cancel{\tfrac{1}{2} \times U(0)} = \tfrac{1}{2} \times 0.80 \times U(100) + \tfrac{1}{2} \times 0.20 \times U(0) + \cancel{\tfrac{1}{2} \times U(0)},$$

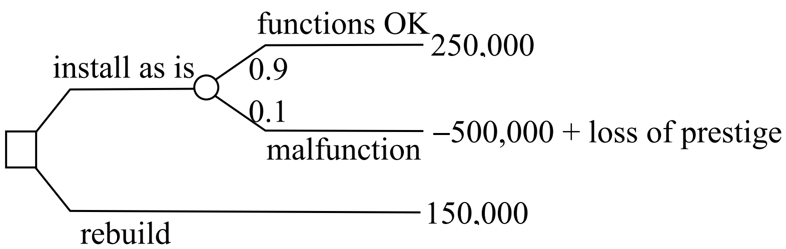
so,

$$\tfrac{1}{2} \times U(70) = \tfrac{1}{2} \times \big(0.80 \times U(100)\big) = \tfrac{1}{2} \times 0.80$$

so,

$$U(70) = 0.80. \quad \square$$

EXERCISE 3.3.8.



First I give a general analysis, and then a simplified.

EU(install as is) = 0.9U(250,000) + 0.1U(−500,000 + loss prestige).

EU(rebuild) = U(150,000).

EU difference is 0.9(U(250,000)–U(150,000)) + 0.1(U(−500,000 + loss prestige) −U(150,000)).

Exceeds 0, meaning that install as is is preferable, iff

$$\frac{U(250{,}000) - U(150{,}000)}{U(150{,}000) - U(-500{,}000 + \text{loss prestige})} \geq 1/9.$$

So, the threshold entails that the loss is nine times worse than the gain.

A simplified analysis results if we set U(−500,000 + loss of prestige) = 0 and U(250,000) = 1. The only unknown now is U(150,000), which is between 0 and 1. Its threshold value is 9/10, the EU of the upper prospect (install as is). □

EXERCISE 3.3.9. This elaboration takes some three pages. Further details are in Appendix A, taking 25 pages.

(a) r = 1.210:

Drill 100% is best. It has CE = 40183.47.

Dril 100% has CE: 40183.47, EU: 2442300.29, normalized EU: 0.104.

Dril 50% has CE: 16391.04, EU: 2077625.25, normalized EU: 0.085.

Override1/8 has CE: 11563.43, EU: 2004911.00, normalized EU: 0.081.

Override1/16 has CE: 15669.17, EU: 2066723.93, normalized EU: 0.084.

Nodrill has CE: 0.00, EU: 1832611.57, normalized EU: 0.072.

(b) r = 0.961.

Drill 100% is best. It has CE = 17543.85.

Dril 100% has CE: 17543.85, EU: 104806.32, normalized EU: 0.124.

Dril 50% has CE: 9376.96, EU: 99892.05, normalized EU: 0.116.

Override1/8 has CE: 11016.87, EU: 100879.61, normalized EU: 0.118.

Override1/16 has CE: 15528.28, EU: 103594.38, normalized EU: 0.122.

Nodrill has CE: 0.00, EU: 94237.49, normalized EU: 0.106.

(c) r = 0.936.

Override 1/16 is best, and has CE = 15514.43. But Dril 100% is very very close.

Dril 100% has CE: 15514.25, EU: 76706.10, normalized EU: 0.127.

Dril 50% has CE: 8735.52, EU: 73761.71, normalized EU: 0.120.

Override1/8 has CE: 10964.05, EU: 74730.56, normalized EU: 0.122.

Override1/16 has CE: 15514.43, EU: 76706.18, normalized EU: 0.127.

Nodrill has CE: 0.00, EU: 69955.43, normalized EU: 0.110.

(d) r = 0.912.

Override 1/16 is best. It has CE = 15501.18.

Dril 100% has CE: 13608.24, EU: 56884.02, normalized EU: 0.129.

Dril 50% has CE: 8130.18, EU: 55144.41, normalized EU: 0.123.

Override1/8 has CE: 10913.68, EU: 56028.99, normalized EU: 0.126.

Override1/16 has CE: 15501.18, EU: 57483.95, normalized EU: 0.131.

Nodrill has CE: 0.00, EU: 52552.72, normalized EU: 0.114.

(e) r = 0.500.

Override 1/16 is best. It has CE = 15281.15.

Dril 100% has CE: −12889.13, EU: 370.28, normalized EU: 0.183.

Dril 50% has CE: −773.51, EU: 386.30, normalized EU: 0.203.

Override1/8 has CE: 10099.80, EU: 400.12, normalized EU: 0.220.

Override1/16 has CE: 15281.15, EU: 406.55, normalized EU: 0.228.

Nodrill has CE: 0.00, EU: 387.30, normalized EU: 0.204.

(f) r = 0.010.

Override 1/16 is best. It has CE = 15036.89.

Dril 100% has CE: −31872.20, EU: 1.1239, normalized EU: 0.279.

Dril 50% has CE: −8298.73, EU: 1.1259, normalized EU: 0.339.

Override1/8 has CE: 9246.81, EU: 1.1273, normalized EU: 0.377.

Override1/16 has CE: 15036.89, EU: 1.1277, normalized EU: 0.389.

Nodrill has CE: 0.00, EU: 1.1266, normalized EU: 0.357.

(g) The following table summarizes the above results.

TABLE Certainty Equivalents of the five prospects for various utility functions.

power r	Drill 100%	Drill 50%	Override 1/8	Override 1/16	Nodril
1.210	**40,183**	16,391	11,563	15,669	0
0.961	**17,544**	9377	11,017	15,528	0
0.936	*15,514.25*	8736	10,964	**15,514.43**	0
0.912	13,608	8130	10,914	**15,501**	0
0.500	−12,889	−774	10,100	**15,281**	0
0.010	−31,872	−8299	9247	**15,037**	0
Winkler	−30,000	−10,000	10,000	**15,000**	0

Approximate indifference results for $r = 0.936$. For higher r, U becomes more risk seeking and drill 100% is more and more preferred, and, for lower r, U becomes more risk averse and override 1/16 is more and preferred. The other options are never preferred. For the smallest $r = 0.01$, the results are close to Winkler's CEs. Given that Override1/16 stays optimal till $r = 0.93$, utility is not a sensitive variable, contrary to suggestions by Winkler (1972, p. 282). His analysis concerns a case where the optimum is so clearly optimal that no variable is sensitive. □

EXERCISE 4.12.2. To discuss the sure-thing principle, we have to consider events. Take any events E_1, E_2, E_3 with probabilities given, with $P(E_1) = .01$, $P(E_2) = .89$, and $P(E_3) = .10$. The table shows event-contingent prospects.

	E_1	E_2	E_3
g^u	0	10×10^6	50×10^6
g^ℓ	10×10^6	10×10^6	10×10^6
h^u	0	0	50×10^6
h^ℓ	10×10^6	0	10×10^6

The prospect g^u corresponds to the Upper prospect in Fig. g, g^ℓ to the Lower prospect in that figure, and h^u and h^ℓ similarly correspond to the prospects in Fig. h. The sure-thing principle requires that a preference between two prospects be independent of common outcomes. Hence, the preference between g^u and g^ℓ should be independent of the outcome under event E_2, and the preference between h^u and h^ℓ should be similarly. If event E_2 is ignored, the g and h prospects become identical and the preference between them should be the same.

In the notation used in the definition of the sure-thing principle in §4.8.1, define $x = g^u$, $y = g^\ell$, $\alpha = 10 \times 10^6$, $\beta = 0$, $E = E_2$. Then the sure-thing principle requires that $[\alpha_E x \succcurlyeq \alpha_E y \;\Leftrightarrow\; \beta_E x \succcurlyeq \beta_E y]$. In other words, $[g^u \succcurlyeq g^\ell \;\Leftrightarrow\; h^u \succcurlyeq h^\ell]$. □

EXERCISE 9.2.1. PT(0.1: 9, 0.3: 1, 0.5: –1, 0.1: –4)

$= 0.01 \times 3.74 + 0.15 \times 1 - 2.25 \times (0.46 \times 1 + 0.32 \times 3.03) = -3.00.$

PT(0.5: 3, 0.5: –2) = $\frac{1}{4} \times 1.93 - 0.71 \times 2.25 \times 1.74 = -2.29$.

Hence, (0.5: 3, 0.5: –2) is preferred. □

EXERCISE 9.3.12.

a) Exercise 3.1.1 found 0.9524 as the switching utility. The answer $p = 0.97$, under an EU-althrough analysis, implies U(artificial speech) = 0.97, exceeding the switching utility. Hence, surgery is chosen.

b) Now U(artificial speech) = $w^+(0.97) \times 1 + (1 - w^+(0.97)) \times 0 = w^+(0.97) = 0.84$.

c) Now the utility U(artificial speech) = 0.84 is below the switching utility 0.9524, and radiotherapy is recommended.

In this exercise it is crucial for the optimal decision what descriptive theory we assume. □

EXERCISE 9.3.13.

We have

EU(radiotherapy) = $0.60 \times U(NV) + 0.16 \times U(AS) + 0.24 \times U(D)$ (**)

EU(surgery) = $0.70 \times U(AS) + 0.09 \times U(AS) + 0.21 \times U(D)$ (***)

(a) $U(AS) = 0.995 \times 1 + 0.005 \times 0 = 0.995$. EU(radiotherapy) = 0.759; EU(surgery) = 0.786. Surgery is chosen.

(b) $U(AS) = w(0.995) \times 1 + (1-w(0.995)) \times 0 = 0.940$.

EU(radiotherapy) = $0.60 \times 1 + 0.16 \times 0.712 + 0.24 \times 0 = 0.750$.

EU(surgery) = $0.70 \times 0.712 + 0.09 \times 0.712 + 0.21 \times 0 = 0.743$.

Now preference is reversed relative to EU, and radiotherapy is chosen, although it is a close call. The corrective procedure is essential!

(c) We now take $U(AS) = u(AS) = 0$. Under PT, we cannot add a constant to U or u, but we can still multiply them by any positive number. Hence, we may assume $u(NV) = U(NV) = 1$. The unknown to be resolved now is U(D). The indifference in Figure 3.1.2 implies

$U(AS) = 0 = w^+(p) \times 1 + w^-(1-p)U(D)$.

$-w^+(p) \times 1 = w^-(1-p)U(D)$.

$$U(D) = -\frac{w^+(p)}{w^-(1-p)} = -\frac{w^+(0.995)}{w^-(0.005)} = -25.252.$$

$u(d) = U(D)/\lambda = -25.252/2.25 = -11.223$.

For EU, we have to decide whether we take U or u as utility function. That is, if we remove loss aversion or keep it. By our definition, loss aversion is irrational, so should be removed. Hence, under EU, we take u and not U as utility function. Then

EU(radiotherapy) = $0.60 \times 1 + 0.16 \times 0 + 0.24 \times -11.223 = -2.094$.

EU(surgery) = $0.70 \times 0 + 0.09 \times 0 + 0.21 \times -11.223 = -2.357$.

Radiotherapy is, again, the preferred option.

Because utility is scaled differently here than in part b, it is not readily visible if the preference here is stronger or weaker. But it can be seen that it is considerably stronger. The utility difference, 0.263, takes a larger part of the total utility range, i.e., total utility of living, (1+11.223 = 12.223) (> 2%), than in b, where it was 0.007 of total utility range 1 (< 1%). Adding loss aversion considerably reinforces the preference for radiotherapy. Loss aversion enhances risk aversion so, correcting for it enhances risk seeking. □

EXERCISE 9.3.14.
If $PT \geq 0$, then
$PT(CE) = CE^{0.88} = PT$;
$CE = PT^{1/0.88} = PT^{1.136}$.

If $PT \leq 0$, then
$PT(CE) = -2.25(-CE)^{0.88} = PT$;
$(-CE)^{0.88} = -PT/2.25$;
$-CE = (-PT/2.25)^{1/0.88} = (-PT/2.25)^{1.136}$;
$CE = -(-PT/2.25)^{1.136}$.
□

EXERCISE 9.5.3. The attitude described is part of intrinsic utility and not of loss aversion, and it is rational. Utility concerns final wealth, and need not be affected by changes in frame or perceived reference point. If we were to change the perceived reference point, there will be a kink of utility not at the newly perceived reference point, but at the final wealth level corresponding with what is the reference point right now. □

EXERCISE 10.3.4. Note that we can use the same rank-ordering for the act and the generated probability distribution. The result follows from substitution and is not elaborated on. □

EXERCISE 10.5.7. In the upper prospect in Fig. g, the rank of event E_2 (having probability 0.89) must be E_3. In the lower prospect in Fig. h, the rank of event E_2 (having probability 0.89) must be the worst (E_3^c). Hence, the rank of the common-outcome event cannot be the same in both choice situations, and the rank-sure-thing principle is not being tested here. □

EXERCISE 11.3.3^b.

(a) The optimism index is 0.

(b) There is ambiguity neutrality in the sense of aversion, and the optimism component gives no manifestation of ambiguity. Yet the sensitivity component can still do so. This even holds if probabilistic sophistication holds within $\mathcal{A}$, i.e. if there exists a subjective probability measure P on $\mathcal{A}$ and a source function $w_{\mathcal{A}}$ such that RDU with weighting function $W(A) = w_{\mathcal{A}}(P(\cdot))$ holds on $\mathcal{A}$. Assume for example that w for risk is the identity, which implies that expected utility holds for $\mathcal{R}$, and that $w_{\mathcal{A}}$ is a symmetric inverse-S weighting function, such as $\dfrac{p^a}{p^a + (1-p)^a}$.(Eq. 7.2.4) with a = 0.69. □

EXERCISE A3.1.

(a) $\theta = 0.495$, with distance 0.0056. $CE(0.36_{0.50}0) = 0.089$. We have $0.36_{0.50}0 \prec 0.10$.

TABLE. Theoretical CEs for $\theta = 0.495$

$0.09_{0.50}0.25$	$0.64_{0.50}0$	$0.16_{0.50}0$	$0.36_{0.50}0$
0.1599	0.1578	0.0394	0.0887

(b) $\theta = 0.482$, and distance 0.0141. $CE(0.36_{0.50}0) = 0.085$. We have $0.36_{0.50}0 \prec 0.10$.

TABLE. Theoretical CEs for $\theta = 0.495$

$0.09_{0.50}0.25$	$0.64_{0.50}0$	$0.16_{0.50}0$	$0.36_{0.50}0$
0.1596	0.1519	0.0380	0.0855

(c) For $\theta = 6.8$, the distance is 0.0139. It is smaller than the distance found in Part (b). $CE(0.36_{0.50}0) = 0.325$. We have $0.36_{0.50}0 \succ 0.10$.

TABLE. Theoretical CEs for $\theta = 0.495$

$0.09_{0.50}0.25$	$0.64_{0.50}0$	$0.16_{0.50}0$	$0.36_{0.50}0$
0.2266	0.5780-	0.1406	0.3252

DISCUSSION. The value $\theta = 0.482$ found in part (b) is only a local optimum. $\theta = 6.8$ of part (c) fits the data better. Every θ larger than 6.8 fits the data even better, and the distance can be made arbitrarily small by taking θ sufficiently large. The reason is that for large θ all utility values become extremely small. For instance, $0.64^{6.8} = 0.048$. Obviously, their differences and all distances then also are extremely small. These θ values imply extreme risk seeking, and their empirical predictions are completely off.

Similar anomalies do not occur for the distance measure used in this book and in Part a. The value $\theta = 0.495$ found there is a global optimum. For large values of θ the distance becomes larger. For $\theta = 6.8$ the distance is 0.26. □

Appendix A. Elaboration of Exercise 3.3.9

The details of this exercise will take 25 pages. Their end will be indicated by two rows of squares.

(a) Calculations for r = 1.21

ANALYSIS OF DRIL100%

EU(dril100%) =

$0.70 \times U(-68000.00) +$

$0.05 \times U(-20000.00) +$

$0.15 \times U(28000.00) +$

$0.05 \times U(412000.00) +$

$0.05 \times U(892000.00)$

=

$0.70 \times 882500.10 +$

$0.05 \times 1541244.20 +$

$0.15 \times 2254281.79 +$

$0.05 \times 9061100.38 +$

0.05×19125814.53

=

617750.07 +

77062.21 +

338142.27 +

453055.02 +

956290.73

=

2442300.29.

EU of drill100% if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.10.

CE of drill 100% =

U^{-1}(EU(dril100%)) =

EU(dril100%)$^{1/r}$ − 150000 =

$2442300.29^{0.83} - 150000$ =

40183.47.

ANALYSIS OF DRIL50%

EU(dril50%) =

$0.70 \times U(-34000.00) +$

$0.05 \times U(-10000.00) +$

$0.15 \times U(14000.00) +$

$0.05 \times U(206000.00) +$

$0.05 \times U(446000.00)$

=

$0.70 \times 1342746.93 +$

$0.05 \times 1685834.45 +$

$0.15 \times 2041554.87 +$

$0.05 \times 5215002.22 +$

0.05×9728546.84

=

939922.85 +

84291.72 +

306233.23 +

260750.11 +

486427.34

=

2077625.25

EU of drill50% if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.08.

CE of drill 50% =

$U^{-1}(EU(dril50\%))$ =

$EU(dril50\%)^{1/r} - 150000$ =

$2077625.25^{0.83} - 150000$ =

16391.04.

ANALYSIS OF OVERR8

EU(overr8) =

$0.70 \times U(0.00)$ +

$0.05 \times U(6000.00)$ +

$0.15 \times U(12000.00)$ +

$0.05 \times U(60000.00)$ +

$0.05 \times U(120000.00)$

=

0.70×1832611.57 +

0.05×1921678.65 +

0.15×2011468.18 +

0.05×2753501.69 +

0.05×3732073.21

=

1282828.10 +

96083.93 +

301720.23 +

137675.08 +

186603.66

=

2004911.00.

EU of override 8 if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.08.

CE of override 1/8 =

$U^{-1}(EU(overr8))$ =

$\mathrm{EU(overr8)}^{1/r} - 150000 =$

$2004911.00^{0.83} - 150000 =$

11563.43.

ANALYSIS OF OVERR16

EU(overr16) =

$0.70 \times \mathrm{U}(10000.00) +$

$0.05 \times \mathrm{U}(13000.00) +$

$0.15 \times \mathrm{U}(16000.00) +$

$0.05 \times \mathrm{U}(40000.00) +$

$0.05 \times \mathrm{U}(70000.00)$

=

$0.70 \times 1981459.40 +$

$0.05 \times 2026501.83 +$

$0.15 \times 2071718.70 +$

$0.05 \times 2439449.64 +$

0.05×2912939.36

=

1387021.58 +

101325.09 +

310757.81 +

121972.48 +

145646.97

=

2066723.93.

EU of override 16 if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.08.

CE of override 1/16 =

$\mathrm{U}^{-1}(\mathrm{EU(overr16)}) =$

$\mathrm{EU(overr16)}^{1/r} - 150000 =$

$2066723.93^{0.83} - 150000 =$

15669.17.

ANALYSIS OF NO DRILLING

EU of not drilling = 1832611.57

EU of nodrill if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.07

CE of Nodril = 0.

(b) Calculations for r = 0.961

ANALYSIS OF DRIL100%

EU(dril100%) =

$0.70 \times U(-68000.00)$ +

$0.05 \times U(-20000.00)$ +

$0.15 \times U(28000.00)$ +

$0.05 \times U(412000.00)$ +

$0.05 \times U(892000.00)$

=

0.70×52744.25 +

0.05×82129.57 +

0.15×111084.54 +

0.05×335348.68 +

0.05×606975.10

=

36920.97 +

4106.48 +

16662.68 +

16767.43 +

30348.76

=

104806.32.

EU of drill100% if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.12.

CE of drill 100% =

U^{-1}(EU(dril100%)) =

EU(dril100%)$^{1/r}$ − 150000 =

104806.32$^{1.04}$ − 150000 =

17543.85.

ANALYSIS OF DRIL50%

EU(dril50%) =

$0.70 \times U(-34000.00) +$

$0.05 \times U(-10000.00) +$

$0.15 \times U(14000.00) +$

$0.05 \times U(206000.00) +$

$0.05 \times U(446000.00)$

=

$0.70 \times 73611.24 +$

$0.05 \times 88191.97 +$

$0.15 \times 102675.05 +$

$0.05 \times 216243.69 +$

0.05×354822.91

=

51527.87 +

4409.60 +

15401.26 +

10812.18 +

17741.15

=

99892.05.

EU of drill50% if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.12.

CE of drill 50% =

U^{-1}(EU(dril50%)) =

EU(dril50%)$^{1/r}$ − 150000 =

$99892.05^{1.04} - 150000$ =

9376.96.

ANALYSIS OF OVERR8

EU(overr8) =

$0.70 \times U(0.00)$ +

$0.05 \times U(6000.00)$ +

$0.15 \times U(12000.00)$ +

$0.05 \times U(60000.00)$ +

$0.05 \times U(120000.00)$

=

0.70×94237.49 +

0.05×97857.19 +

0.15×101471.46 +

0.05×130212.52 +

0.05×165783.22

=

65966.24 +

4892.86 +

15220.72 +

6510.63 +

8289.16

=

100879.61.

EU of override 8 if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.12.

CE of override 1/8 =

$U^{-1}(EU(overr8)) =$

$EU(overr8)^{1/r} - 150000 =$

$100879.61^{1.04} - 150000 =$

11016.87.

ANALYSIS OF OVERR16

$EU(overr16) =$

$0.70 \times U(10000.00) +$

$0.05 \times U(13000.00) +$

$0.15 \times U(16000.00) +$

$0.05 \times U(40000.00) +$

$0.05 \times U(70000.00)$

=

$0.70 \times 100267.30 +$

$0.05 \times 102073.33 +$

$0.15 \times 103878.07 +$

$0.05 \times 118272.07 +$

0.05×136165.85

=

70187.11 +

5103.67 +

15581.71 +

5913.60 +

6808.29

=

103594.38.

EU of override 16 if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.12.

CE of override 1/16 =

$U^{-1}(EU(overr16)) =$

$EU(overr16)^{1/r} - 150000 =$

$103594.38^{1.04} - 150000 =$

15528.28.

ANALYSIS OF NO DRILLING

EU of not drilling = 94237.49.

EU of nodrill if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.11.

(c) Calculations for r = 0.936

ANALYSIS OF DRIL100%

EU(dril100%) =

$0.70 \times U(-68000.00) +$

$0.05 \times U(-20000.00) +$

$0.15 \times U(28000.00) +$

$0.05 \times U(412000.00) +$

$0.05 \times U(892000.00)$

=

$0.70 \times 39749.33 +$

$0.05 \times 61185.85 +$

$0.15 \times 82109.45 +$

$0.05 \times 240853.63 +$

0.05×429263.72

=

27824.53 +

3059.29 +

12316.42 +

12042.68 +

21463.19

=

76706.10.

EU of drill100% if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.13.

CE of drill 100% =

U^{-1}(EU(dril100%)) =

EU(dril100%)$^{1/r}$ − 150000 =

76706.10$^{1.07}$ − 150000 =

15514.25.

ANALYSIS OF DRIL50%

EU(dril50%) =

0.70 × U(−34000.00) +

0.05 × U(−10000.00) +

0.15 × U(14000.00) +

0.05 × U(206000.00) +

0.05 × U(446000.00)

=

0.70 × 54996.20 +

0.05 × 65580.67 +

0.15 × 76049.06 +

0.05 × 157093.13 +

0.05 × 254466.43

=

38497.34 +

3279.03 +

11407.36 +

7854.66 +

12723.32

=

73761.71.

EU of drill50% if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.12.

CE of drill 50% =

U^{-1}(EU(dril50%)) =

EU(dril50%)$^{1/r}$ − 150000 =

73761.71$^{1.07}$ − 150000 =

8735.52.

ANALYSIS OF OVERR8

EU(overr8) =

$0.70 \times$ U(0.00) +

$0.05 \times$ U(6000.00) +

$0.15 \times$ U(12000.00) +

$0.05 \times$ U(60000.00) +

$0.05 \times$ U(120000.00)

=

0.70×69955.43 +

0.05×72571.25 +

0.15×75180.65 +

0.05×95851.14 +

0.05×121270.87

=

48968.80 +

3628.56 +

11277.10 +

4792.56 +

6063.54

=

74730.56.

EU of override 8 if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.12.

CE of override 1/8 =
U^{-1}(EU(overr8)) =
EU(overr8)$^{1/r}$ − 150000 =
74730.56$^{1.07}$ − 150000 =
10964.05.

ANALYSIS OF OVERR16
EU(overr16) =
$0.70 \times U(10000.00)$ +
$0.05 \times U(13000.00)$ +
$0.15 \times U(16000.00)$ +
$0.05 \times U(40000.00)$ +
$0.05 \times U(70000.00)$
=
0.70×74311.55 +
0.05×75614.94 +
0.15×76916.79 +
0.05×87279.73 +
0.05×100116.95
=
52018.08 +
3780.75 +
11537.52 +
4363.99 +
5005.85
=
76706.18.

EU of override 16 if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.13.

CE of override 1/16 =

$U^{-1}(EU(overr16))$ =

$EU(overr16)^{1/r} - 150000$ =

$76706.18^{1.07} - 150000$ =

15514.43.

ANALYSIS OF NO DRILLING

EU of not drilling = 69955.43.

EU of nodrill if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.11.

(d) Calculations for r = 0.912

ANALYSIS OF DRIL100%

EU(dril100%) =

$0.70 \times U(-68000.00)$ +

$0.05 \times U(-20000.00)$ +

$0.15 \times U(28000.00)$ +

$0.05 \times U(412000.00)$ +

$0.05 \times U(892000.00)$

=

0.70×30296.90 +

0.05×46122.87 +

0.15×61430.36 +

0.05×175290.96 +

0.05×307818.96

=

21207.83 +

2306.14 +

9214.55 +

8764.55 +

15390.95

=

56884.02.

EU of drill100% if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.13

CE of drill 100% =

U^{-1}(EU(dril100%)) =

EU(dril100%)$^{1/r}$ − 150000 =

56884.02$^{1.10}$ − 150000 =

13608.24.

ANALYSIS OF DRIL50%

EU(dril50%) =

$0.70 \times U(-34000.00)$ +

$0.05 \times U(-10000.00)$ +

$0.15 \times U(14000.00)$ +

$0.05 \times U(206000.00)$ +

$0.05 \times U(446000.00)$

=

0.70×41570.54 +

0.05×49347.91 +

0.15×57008.23 +

0.05×115590.57 +

0.05×184937.33

=

29099.38 +

2467.40 +

8551.24 +

5779.53 +

9246.87

=

55144.41.

EU of drill50% if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.12.

CE of drill 50% =

U^{-1}(EU(dril50%)) =

EU(dril50%)$^{1/r}$ − 150000 =

55144.41$^{1.10}$ − 150000 =

8130.18.

ANALYSIS OF OVERR8

EU(overr8) =

$0.70 \times$ U(0.00) +

$0.05 \times$ U(6000.00) +

$0.15 \times$ U(12000.00) +

$0.05 \times$ U(60000.00) +

$0.05 \times$ U(120000.00)

=

0.70×52552.72 +

0.05×54466.52 +

0.15×56373.85 +

0.05×71427.26 +

0.05×89826.35

=

36786.91 +

2723.33 +

8456.08 +

3571.36 +

4491.32

=

56028.99.

EU of override 8 if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.13.

CE of override 1/8 =

U^{-1}(EU(overr8)) =

EU(overr8)$^{1/r}$ − 150000 =

56028.99$^{1.10}$ − 150000 =

10913.68.

ANALYSIS OF OVERR16

EU(overr16) =

$0.70 \times$ U(10000.00) +

$0.05 \times$ U(13000.00) +

$0.15 \times$ U(16000.00) +

$0.05 \times$ U(40000.00) +

$0.05 \times$ U(70000.00)

=

0.70×55738.78 +

0.05×56691.13 +

0.15×57641.94 +

0.05×65196.35 +

0.05×74522.86

=

39017.14 +

2834.56 +

8646.29 +

3259.82 +

3726.14

=

57483.95.

EU of override 16 if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.13.

CE of override 1/16 =
U^{-1}(EU(overr16)) =
$\text{EU(overr16)}^{1/r} - 150000$ =
$57483.95^{1.10} - 150000$ =
15501.18.

ANALYSIS OF NO DRILLING
EU of not drilling = 52552.72.
EU of nodrill if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.11.

(e) Calculations for r = 0.500

ANALYSIS OF DRIL100%
EU(dril100%) =
$0.70 \times U(-68000.00)$ +
$0.05 \times U(-20000.00)$ +
$0.15 \times U(28000.00)$ +
$0.05 \times U(412000.00)$ +
$0.05 \times U(892000.00)$
=
0.70×286.36 +
0.05×360.56 +

$0.15 \times 421.90 +$

$0.05 \times 749.67 +$

0.05×1020.78

$=$

$200.45 +$

$18.03 +$

$63.29 +$

$37.48 +$

51.04

$=$

$370.28.$

EU of drill100% if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.18.

CE of drill 100% =

$U^{-1}(EU(dril100\%)) =$

$EU(dril100\%)^{1/r} - 150000 =$

$370.28^{2.00} - 150000 =$

$-12889.13.$

ANALYSIS OF DRIL50%

$EU(dril50\%) =$

$0.70 \times U(-34000.00) +$

$0.05 \times U(-10000.00) +$

$0.15 \times U(14000.00) +$

$0.05 \times U(206000.00) +$

$0.05 \times U(446000.00)$

$=$

$0.70 \times 340.59 +$

$0.05 \times 374.17 +$

$0.15 \times 404.97 +$

0.05×596.66 +

0.05×772.01

=

238.41 +

18.71 +

60.75 +

29.83 +

38.60

=

386.30

EU of drill50% if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.20.

CE of drill 50% =

U^{-1}(EU(dril50%)) =

EU(dril50%)$^{1/r}$ − 150000 =

$386.30^{2.00}$ − 150000 =

−773.51.

ANALYSIS OF OVERR8

EU(overr8) =

$0.70 \times$ U(0.00) +

$0.05 \times$ U(6000.00) +

$0.15 \times$ U(12000.00) +

$0.05 \times$ U(60000.00) +

$0.05 \times$ U(120000.00)

=

0.70×387.30 +

0.05×394.97 +

0.15×402.49 +

0.05×458.26 +

0.05×519.62

=

271.11 +

19.75 +

60.37 +

22.91 +

25.98

=

400.12.

EU of override 8 if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.22.

CE of override 1/8 =

U^{-1}(EU(overr8)) =

EU(overr8)$^{1/r}$ − 150000 =

$400.12^{2.00}$ − 150000 =

10099.80.

ANALYSIS OF OVERR16

EU(overr16) =

$0.70 \times$ U(10000.00) +

$0.05 \times$ U(13000.00) +

$0.15 \times$ U(16000.00) +

$0.05 \times$ U(40000.00) +

$0.05 \times$ U(70000.00)

=

0.70×400.00 +

0.05×403.73 +

0.15×407.43 +

0.05×435.89 +

0.05×469.04

=

280.00 +

20.19 +

61.11 +

21.79 +

23.45

=

406.55.

EU of override 16 if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.23.

CE of override 1/16 =

U^{-1}(EU(overr16)) =

EU(overr16)$^{1/r}$ − 150000 =

406.55$^{2.00}$ − 150000 =

15281.15.

ANALYSIS OF NO DRILLING

EU of not drilling = 387.30.

EU of nodrill if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.20.

(f) Calculations for r = 0.010

ANALYSIS OF DRIL100%

EU(dril100%) =

0.70 × U(−68000.00) +

0.05 × U(−20000.00) +

0.15 × U(28000.00) +

$0.05 \times U(412000.00) +$

$0.05 \times U(892000.00)$

$=$

$0.70 \times 1.12 +$

$0.05 \times 1.12 +$

$0.15 \times 1.13 +$

$0.05 \times 1.14 +$

0.05×1.15

$=$

$0.78 +$

$0.06 +$

$0.17 +$

$0.06 +$

0.06

$=$

$1.1239.$

EU of drill100% if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.2793.

CE of drill 100% =

$U^{-1}(EU(dril100\%)) =$

$EU(dril100\%)^{1/r} - 150000 =$

$1.12^{100} - 150000 =$

$-31872.20.$

ANALYSIS OF DRIL50%

$EU(dril50\%) =$

$0.70 \times U(-34000.00) +$

$0.05 \times U(-10000.00) +$

$0.15 \times U(14000.00) +$

$0.05 \times U(206000.00) +$

$0.05 \times U(446000.00)$

$=$

$0.70 \times 1.12 +$

$0.05 \times 1.13 +$

$0.15 \times 1.13 +$

$0.05 \times 1.14 +$

0.05×1.14

$=$

$0.79 +$

$0.06 +$

$0.17 +$

$0.06 +$

0.06

$=$

1.1259.

EU of drill50% if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.3387.

CE of drill 50% =

$U^{-1}(EU(dril50\%)) =$

$EU(dril50\%)^{1/r} - 150000 =$

$1.13^{100} - 150000 =$

−8298.73.

ANALYSIS OF OVERR8

EU(overr8) =

$0.70 \times U(0.00) +$

$0.05 \times U(6000.00) +$

$0.15 \times U(12000.00) +$

$0.05 \times U(60000.00) +$

$0.05 \times U(120000.00)$

$=$

0.70×1.13 +

0.05×1.13 +

0.15×1.13 +

0.05×1.13 +

0.05×1.13

=

0.79 +

0.06 +

0.17 +

0.06 +

0.06

=

1.1273.

EU of override 8 if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.3769.

CE of override 1/8 =

U^{-1}(EU(overr8)) =

$EU(overr8)^{1/r} - 150000$ =

$1.13^{100} - 150000$ =

9246.81.

ANALYSIS OF OVERR16

EU(overr16) =

$0.70 \times U(10000.00)$ +

$0.05 \times U(13000.00)$ +

$0.15 \times U(16000.00)$ +

$0.05 \times U(40000.00)$ +

$0.05 \times U(70000.00)$

=

0.70×1.13 +

0.05×1.13 +

$0.15 \times 1.13 +$

$0.05 \times 1.13 +$

0.05×1.13

$= .79 +$

$0.06 +$

$0.17 +$

$0.06 +$

0.06

$=$

$1.1277.$

EU of override 16 if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.3886.

CE of override 1/16 =

$U^{-1}(EU(overr16)) =$

$EU(overr16)^{1/r} - 150000 =$

$1.13^{100} - 150000 =$

15036.89.

ANALYSIS OF NO DRILLING

EU of not drilling = 1.1266.

EU of nodrill if U is normalized at −100,000 and 900,000, as in Winkler (1972), is 0.3573.

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