

A Correction of Tversky, Amos & Peter P. Wakker (1995), "Risk Attitudes and Decision Weights," *Econometrica* 63, 1255–1280.

By Peter Wakker,
CentER, Tilburg University, The Netherlands
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The definitions of bounded subadditivity in the paper are formulated in a misleading manner. Only footnotes 7 and 10 suggest the right definitions. Let me first repeat the, incorrect, definition as stated in the paper for decision under risk:

w satisfies *bounded subadditivity*, or *subadditivity (SA)* for short, if there exist constants $\varepsilon \geq 0$ and $\varepsilon' \geq 0$ such that:

$$(4.1) \quad w(q) \geq w(p+q) - w(p) \quad \text{whenever } p+q \leq 1-\varepsilon$$

and

$$(4.2) \quad 1 - w(1-q) \geq w(p+q) - w(p) \quad \text{whenever } p \geq \varepsilon'.$$

This definition is not correct. Literally taken, according to the rules of mathematics, **every** weighting function w satisfies this condition! The reason is that for every w there exist such ε and ε' : $\varepsilon = \varepsilon' = 1$! With those ε and ε' , (4.1) and (4.2) should only hold for $p+q \leq 0$ and $p \geq 1$, respectively, and then these conditions are satisfied for every w whatsoever. Hence every w satisfies conditions (4.1) and (4.2). That was, of course, not our intention. The correct definition should express that bounded SA depends on the boundary constants ε and ε' chosen. So the correct definition is:

For constants $\varepsilon \geq 0$ and $\varepsilon' \geq 0$, w satisfies (*bounded*) *subadditivity* with respect to ε and ε' ($\varepsilon, \varepsilon'$ -SA), if:

$$(4.1) \quad w(q) \geq w(p+q) - w(p) \quad \text{whenever } p+q \leq 1-\varepsilon$$

and

$$(4.2) \quad 1 - w(1-q) \geq w(p+q) - w(p) \quad \text{whenever } p \geq \varepsilon'.$$

If the choice of the constants ε and ε' is well-understood, they are suppressed and we simply say bounded SA or SA.

Footnote 7 in the paper suggests, correctly, that the concepts should depend on the boundary constants ε and ε' chosen, but is not very clear on that. I apologize for the confusions caused by our unfortunate definition. An important question in applications is now, of course, how the boundary constants are to be chosen. Usually and empirically the choice $\varepsilon = \varepsilon' = 0.10$ seems to be fine so these choices are recommended.

A similar problem occurs for uncertainty. Let me first repeat the, incorrect, definition as stated in the paper:

W satisfies *bounded subadditivity*, or *subadditivity (SA)* for short, if there are events E, E' such that

$$(5.1) \quad W(B) \geq W(A \cup B) - W(A) \quad \text{whenever } W(A \cup B) \leq W(S-E);$$

and

$$(5.2) \quad 1 - W(S-B) \geq W(A \cup B) - W(A) \quad \text{whenever } W(A) \geq W(E').$$

This definition is, again, not correct. **Every** weighting function W satisfies this condition. For every W there exist such E and E' : $E = E' = S$. The correct definition should express that bounded SA depends on the boundary events E and E' chosen. So the correct definition is:

For events E and E' , W satisfies (*bounded*) *subadditivity* with respect to E and E' (E, E' -SA), if:

$$(5.1) \quad W(B) \geq W(A \cup B) - W(A) \quad \text{whenever } W(A \cup B) \leq W(S - E);$$

and

$$(5.2) \quad 1 - W(S - B) \geq W(A \cup B) - W(A) \quad \text{whenever } W(A) \geq W(E').$$

If the choice of the events E and E' is well-understood, they are suppressed and we simply say bounded SA or SA.

In all other places in the paper the notions defined should similarly depend on the boundary constants or events chosen. This holds for conditions (4.3) and (4.4), (5.3) and (5.4), (6.1) and (6.2), (6.3) and (6.4), (6.5) and (6.6), (6.8) and (6.9), (7.1) and (7.2), and (7.3) and (7.4). At those places the role of boundary constants/events was never again formulated as incorrectly as in the conditions displayed above. Instead, the issue was avoided by using ambiguous formulations which is, of course, neither desirable.

Let me end by apologizing once again for the inconveniences caused.

Peter Wakker