SUBJECTIVE EXPECTED UTILITY WITH NONINCREASING RISK AVERSION

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Abstract

It is shown that assumptions about risk aversion, usually studied under the presupposition of expected utility maximization, have a surprising extra merit at an earlier stage of the measurement work: together with the sure-thing principle, these assumptions imply subjective expected utility maximization for monotonic continuous weak orders.

1.   Introduction

There has not yet been obtained a characterization of subjective expected utility maximization in the literature that is fully satisfactory in the sense of using necessary and sufficient conditions that are all empirically meaningful and have a clear intuitive meaning. The main nonnecessary condition used by Savage [7] is his P6, a kind of continuity condition, requiring some structure on the state space, which must be "fine". The main nonnecessary condition used in Wakker [9—11] is a continuity condition, requiring structure on the consequence space, which also must be "fine" in some sense.

Because of the frequent use of subjective expected utility models and all kinds of special forms of these, further "derivations" seem desirable. A *derivation* of a (specified form of a) subjective expected utility model gives a list of conditions sufficient for the applicability of the model. It is desirable that the list be as short as possible, with as many as possible behaviourally meaningful conditions (such as Savage's P3, the "sure-thing principle", and his P4, ensuring the existence of a "more probable than" relation; or Wakker's "cardinal coordinate independence", a strengthening of the sure-thing principle), with few technical conditions (such as the "Archimedean

axioms" in Krantz et al. [5], see Adams et al. [1]), and with as few conditions as possible that have an unclear empirical status (such as P6 in Savage [7], and continuity in Wakker [9−11]).

In this paper we shall restrict attention to preference relations of decision makers who face uncertainty about the *monetary* consequences of possible actions. We shall derive, for finite state spaces, the subjective expected utility model with risk aversion and nonincreasing (or nondecreasing; or constant; or constant relative) risk aversion. It will be shown that these assumptions on risk aversion simplify the derivation of subjective expected utility maximization by making superfluous Savage's P4 or Wakker's cardinal coordinate independence strengthening of the sure-thing principle. The only nonbehavioural and nonnecessary condition that we shall use is continuity of the preference relation, amounting to continuity of the utility function.

The proof of our main result, theorem 3.3, is based on the adaption of theorems of Pratt [6] to the case of continuous (not necessarily differentiable) utility functions, and to the context of decision making under uncertainty (without all probability distributions available), as it has been given in Wakker et al. [12]; on a theorem of Debreu [2] for additively decomposable representing functions; and on a theorem of Debreu and Koopmans [3] on differentiability properties of additively decomposable quasiconvex functions.

Subsequent theorems use a result of Stehling [8] on additively decomposable homothetic functions.

2.   Decision making under uncertainty

Let $S = \{s_1, \ldots, s_n\}$ denote a finite *state space*, elements of which are (*possible*) *states* (*of nature*). Exactly one is the true state, the others are untrue. By $\mathscr{C}$ we denote a nondegenerate interval, i.e. a convex subset of $\mathbb{R}$ with more than one, so uncountably many, elements. Elements of $\mathscr{C}$ are *consequences*, and are interpreted as amounts of money. Elements of $\mathscr{C}^n$ are called *acts*. An act $x = (x_1, \ldots, x_n)$ yields consequence x_j if state j is the true state.

Let $\succeq$ be a binary relation on $\mathscr{C}^n$ that denotes the *preference relation* of a decision maker who is uncertain about which state of nature is the true one. We write $x \precsim y$ for $y \succeq x$, $x \approx y$ for $[x \succeq y$ and $y \succeq x]$, $x \succ y$ for $[x \succeq y$ and not $y \succeq x]$, $x \prec y$ for $y \succ x$. We call $\succeq$ a *weak order* if it is *complete* ($x \succeq y$ or $y \succeq x$ for all x, y) and *transitive*. Further, $\succeq$ is *continuous* if $\{x : x \succeq y\}$ and $\{x : x \precsim y\}$ are closed for all y, and $\succeq$ is *convex* if $\{x : x \succeq y\}$ is convex for every y. In this paper, $\succeq$ will always be *strongly monotonic*, i.e. $x \succ y$ whenever $x_j \geqslant y_j$ for all j, and $x \neq y$. This implies that no state of nature is considered impossible, and that all coordinates are "essential" (see definition 2.8), i.e. have influence on the preference relation.

A function $V : \mathscr{C}^n \to \mathbb{R}$ *represents* $\succeq$ if $[x \succeq y \Leftrightarrow V(x) \geqslant V(y)]$ for all $x, y \in \mathscr{C}^n$. V is *additively decomposable* if there exist $V_j : \mathscr{C} \to \mathbb{R}$, $j = 1, \ldots, n$,

such that $V(x) = \Sigma V_j(x_j)$ for all x. Such V_j's are called *additive value functions* (*for* $\succeq$) if V also represents $\succeq$.

NOTATION 2.1

For $1 \leqslant i \leqslant n$, $x \in \mathscr{C}^n$, $\alpha \in \mathscr{C}$, $x_{-i}\alpha$ is (x with x_i replaced by α). If further $1 \leqslant j \leqslant n$, $j \neq i$, $\beta \in \mathscr{C}$, then $(x_{-i,j}\alpha, \beta)$ is (x with x_i replaced by α, x_j by β).

NOTATION 2.2

For $\alpha \in \mathscr{C}$, $\overline{\alpha}$ is $(\alpha, \alpha, \ldots, \alpha)$.

DEFINITION 2.3

We say $\succeq$ satisfies the *sure-thing principle* if for all $x, y \in \mathscr{C}^n$, $1 \leqslant i \leqslant n$, $\alpha, \beta \in \mathscr{C}$:

$$x_{-i}\alpha \succeq y_{-i}\alpha \Leftrightarrow x_{-i}\beta \succeq y_{-i}\beta.$$

One can see that $\succeq$ satisfies the sure-thing principle if and only if: $[x \succeq y \Leftrightarrow v \succeq w]$ whenever for each i either $[x_i = v_i$ and $y_i = w_i]$ or $[x_i = y_i$ and $v_i = w_i]$. This latter condition is derived from the sure-thing principle by (finiteness of S and) substitution, one by one, of v_i and w_i for x_i and y_i, for all those i for which $x_i = y_i$, $v_i = w_i$, and by application of the sure-thing principle after every substitution. The second formulation of the sure-thing principle is the one used in Savage [7].

DEFINITION 2.4

We call $[\mathscr{C}^n, \succeq, (p_j)_{j=1}^n, U]$ a *subjective expected utility* (*SEU*) *model* (*for* $\succeq$) if the p_j's are nonnegative real numbers, summing to 1, and U is a function from $\mathscr{C}$ to $\mathbb{R}$, such that $\succeq$ is represented by the function $V : \mathscr{C}^n \to \mathbb{R}$, defined by $V(x) := \Sigma_{j=1}^n p_j U(x_j)$ for all x.

We then call p_j the (*subjective*) *probability* for state s_j, U the (*subjective*) *utility function*, and $V(x)$ the (*subjective*) *expected utility* of x.

For a given SEU model $[\mathscr{C}^n, \succeq, (p_j)_{j=1}^n, U]$, $\succeq$ is *risk averse* if, for all $x \in \mathscr{C}^n$:

$$x \precsim \left(\sum_{j=1}^n p_j x_j, \ldots, \sum_{j=1}^n p_j x_j \right) . \tag{2.1}$$

It should be noted that this property requires the probabilities to be known. We have the following well-known characterization:

THEOREM 2.5

Let $[\mathscr{C}^n, \succeq, (p_j)_{j=1}^n, U]$ be an SEU model for $\succeq$, with $n \geqslant 2$, U continuous, and $\succeq$ strictly monotonic. Then the following three statements are equivalent:

(i)   "U is concave".

(ii)   "$\succeq$ is risk averse".

(iii)   "$\succeq$ is convex".

The properties introduced below usually are considered only in the presence of risk aversion. We prefer to introduce them in general.

DEFINITION 2.6

The preference relation $\succeq$ has *nonincreasing* [respectively, *nondecreasing*; or *constant*] (*absolute*) *risk aversion* if for all $\epsilon \geqslant 0$ [respectively, $\epsilon \leqslant 0$; or $\epsilon \in IR$], and for all x, $x + \bar{\epsilon}$ in $\mathscr{C}^n$, $\alpha, \alpha + \epsilon \in \mathscr{C}$, we have:

$$x \succeq \bar{\alpha} \Rightarrow x + \bar{\epsilon} \succeq \bar{\alpha} + \bar{\epsilon}.$$

DEFINITION 2.7

The preference relation $\succeq$ has *constant relative risk aversion* if for all $\lambda \in IR_{++}$, x, $\lambda x \in \mathscr{C}^n$, $\alpha, \lambda\alpha \in \mathscr{C}$, we have:

$$x \succeq \bar{\alpha} \Rightarrow \lambda x \succeq \lambda\bar{\alpha}.$$

The latter property, while conceptually different, can formally be identified with the condition of "homotheticity" from consumer demand theory. The following property will be used only in reference to other papers:

DEFINITION 2.8

Coordinate (or index) i is *essential* if there exist x, α such that $x_{-i}\alpha \succ x$.

3.   Subjective expected utility for nonincreasing or nondecreasing risk aversion

In the lemma below, $V^r(\alpha)$ [respectively, $V^\ell(\alpha)$; or $V'(\alpha)$] denotes the right-derivative [respectively, the left-derivative; or the derivative] in α of a function V.

LEMMA 3.1

Let $n \geqslant 2$. Let $V_1, \ldots, V_n$ be continuous strictly increasing additive value functions for $\succeq$. Let $\succeq$ be convex, and have nonincreasing risk aversion. Let $\alpha \in \text{int}(\mathscr{C})$, $\beta \in \text{int}(\mathscr{C})$, $\alpha > \beta$. Let $i \neq j$. Then:

$$V_i^r(\alpha) V_j^\ell(\beta) \geqslant V_j^\ell(\alpha) V_i^r(\beta) \tag{3.1}$$

whenever the derivatives are defined.

Proof

We take $(\sigma^k)_{k=1}^\infty$, $(\tau^k)_{k=1}^\infty \in IR_{++}$ such that: $\sigma^k \downarrow 0$, $\tau^k \downarrow 0$, $\alpha + \tau_k$ and $\beta - \sigma_k$ are in $\mathscr{C}$ for all k, $V_i(\beta + \tau^k) - V_i(\beta) = V_j(\beta) - V_j(\beta - \sigma_k)$ for all k. Such σ^k and τ^k exist by continuity and strict increasingness of V_i and V_j. For all k, it follows that

$$(\bar{\beta}_{-i,j}(\beta + \tau^k), (\beta - \sigma^k)) \approx \bar{\beta}.$$

By nonincreasing risk aversion, it follows that:

$$(\bar{\alpha}_{-i,j}(\alpha + \tau^k), (\alpha - \sigma^k)) \succeq \bar{\alpha};$$

i.e.,

$$V_i(\alpha + \tau^k) - V_i(\alpha) \geqslant V_j(\alpha) - V_j(\alpha - \sigma^k)$$

for all k. We obtain:

$$V_i^r(\alpha) V_j^\ell(\beta) = \lim_{k \to \infty} [V_i(\alpha + \tau^k) - V_i(\alpha)] [V_j(\beta) - V_j(\beta - \sigma^k)] / \tau^k \sigma^k$$

$$\geqslant \lim_{k \to \infty} [V_j(\alpha) - V_j(\alpha - \sigma^k)] [V_i(\beta + \tau^k) - V_i(\beta)] / \sigma^k \tau^k = V_j^\ell(\alpha) V_i^r(\beta). \qquad \square$$

REMARK 3.2

If one replaces nonincreasing risk aversion in the above lemma by nondecreasing risk aversion, then

$$V_i^r(\alpha) V_j^\ell(\beta) \leqslant V_j^\ell(\alpha) V_i^r(\beta). \tag{3.2}$$

The proof is analogous to the one above.

We are now ready for our main theorem.

THEOREM 3.3

Let $n \geq 3$. The following three statements are equivalent for the nondegenerate interval $\mathscr{C}$ and the preference relation $\succeq$ on $\mathscr{C}^n$:

(i)   "There exists an SEU model $[\mathscr{C}^n, \succeq, (p_j)_{j=1}^n, U]$ for $\succeq$, with all $p_j > 0$, and with U continuous, strictly increasing, concave; furthermore, for all $\alpha \geq \beta \geq \gamma > \delta$ in $\mathscr{C}$, the function

$$\epsilon \mapsto [U(\alpha + \epsilon) - U(\beta + \epsilon)] / [U(\gamma + \epsilon) - U(\delta + \epsilon)] \tag{3.3}$$

is nondecreasing on its domain".

(ii) "There exists an SEU model $[\mathscr{C}^n, \succeq, (p_j)_{j=1}^n, U]$ for $\succeq$, with all $p_j > 0$, U continuous, strictly increasing. Further, $\succeq$ is risk averse, and has nonincreasing risk aversion".

(iii) "The preference relation $\succeq$ is a continuous strongly monotonic weak order, it satisfies the sure-thing principle, is convex, and has nonincreasing risk aversion".

Proof

Part of the equivalence of (i) and (ii) follows from theorem 2.5. We concentrate on the remainder of the equivalence of (i) and (ii). This does not follow from Pratt ([6], mainly (e) in theorem 1), firstly because his U is assumed twice continuously differentiable, and secondly because his context is decision making under risk, with all probability distributions available, whereas we have only a fixed and finite number of probabilities $p_1, \ldots, p_n$ available. The only result in the literature known to us from which (i) $\Leftrightarrow$ (ii) can be derived is Wakker et al. ([12], theorem 4.1 and lemma A.7.4), mainly by comparing $\succeq$ with $\succeq'$, defined by $x \succeq' y$ if $x - \overline{\epsilon} \succeq y - \overline{\epsilon}$, on an appropriate domain. For brevity, we omit elaboration.

The implication (ii) $\Rightarrow$ (iii) is straightforward, so we finally assume (iii) and derive (ii).

By strong monotonicity, all (so at least three) coordinates are essential. Since $\succeq$ is a continuous weak order, satisfying the sure-thing principle, theorem 3 of Debreu [2] implies that there exist continuous additive value functions $(V_j)_{j=1}^n$ for $\succeq$. Further, we may assume that for some arbitrary fixed $\mu^0 \in \mathscr{C}$, $V_j(\mu^0) = 0$ for all j. By strong monotonicity, the V_j's are strictly increasing. The main problem will now be to show:

The V_i's are proportional.

To see this, first we note that the function V, assigning $\sum_{j=1}^n V_j(x_j)$ to every x, represents a convex $\succeq$, and hence is quasiconcave. It now follows easily from the n-dimen-

sional analogue of theorem 3 in Debreu and Koopmans [3] that, for all i, on int$(\mathscr{C})$: V_i^r and V_i^ϱ are defined and finite, V_i^r is continuous on the right, V_i^ϱ is continuous on the left, and at every point outside a countable set E_i, V_i is differentiable.

Let $E = \cup E_i$, so E is countable. On $[\text{int}(\mathscr{C})] \setminus E$, all V_i are differentiable. Hence, two applications of lemma 3.1 give, for all i, j:

$$V_i^{'}(\alpha) V_j^{'}(\beta) = V_j^{'}(\alpha) V_i^{'}(\beta) \quad \text{for all } \alpha, \beta \in [\text{int}(\mathscr{C})] \setminus E.$$

It follows from this that there exist j and real numbers $(\sigma_i)_{i=1}^n$ such that $V_i^{'} = \sigma_i V_j^{'}$ on $[\text{int}(\mathscr{C})] \setminus E$ for all i. Since E is countable and V_i^r, V_j^r are continuous on the right everywhere on int$(\mathscr{C})$, $V_i^r = \sigma_i V_j^r$ follows on all of int$(\mathscr{C})$; analogously, $V_i^\varrho = \sigma_i V_j^\varrho$. Hence, $V_i - \sigma_i V_j$ has derivative 0 on int$(\mathscr{C})$, and so is constant. Since $V_i(\mu^0) = 0 = V_j(\mu^0)$, the constant is zero, so continuity yields $V_i = \sigma_i V_j$ on all of $\mathscr{C}$, for all i:

The V_i's indeed are proportional.

By strict increasingness of the V_i's, all σ_i's are positive. We define $U := (\sum_{k=1}^n \sigma_k) V_j$ and $p_i := \sigma_i / \sum_{k=1}^n \sigma_k$. This gives an SEU model for $\succeq$. Since, by theorem 2.5, convexity of $\succeq$ implies risk aversion, everything of (ii) follows. $\square$

In (iii) above, we have given a characterization of the quantitative model in (i), completely in terms of properties directly in terms of the preference relation $\succeq$. Hence, we did not use the property of risk aversion in (iii), as this needs the probabilities in its definition. In the context of decision making under uncertainty (unlike the context of decision making under risk), probabilities are not primitives.

One can replace nonincreasing risk aversion by nondecreasing risk aversion in (ii) and (iii) above if one replaces nondecreasingness of the function defined in (3.3) by nonincreasingness. Analogously, one can of course substitute "constant risk aversion" in (ii) and (iii), and constantness of the function defined in (3.3). In the latter case, either U is affine or exponential, as can be derived from theorem 3.5 in the sequel. Finally, if $\mathscr{C} = \mathbb{R}_{++}$, one can replace "nonincreasing risk aversion" in (ii) and (iii) above by "constant relative risk aversion" if one replaces the statement on the function defined in (3.3) by the statement that $U: \alpha \mapsto \log \alpha$, or $U: \alpha \mapsto \lambda \alpha^\rho$, as can be derived from theorem 3.4 in the sequel.

For theorem 3.3, we do not need the assumption of concavity in (i), of risk aversion in (ii), and of convexity in (iii) if $\mathscr{C} = \mathbb{R}$ and we have constant absolute risk aversion, or if $\mathscr{C} = \mathbb{R}_{++}$ and we have constant relative risk aversion. First, we give the latter result, this being directly derivable from Stehling [8].

THEOREM 3.4

Let $n \geqslant 3$. Let $\mathscr{C} = I\!R_{++}$. The following two statements are equivalent for the preference relation $\succeq$ on $\mathscr{C}^n$:

(i) "There exists an SEU model $[\mathscr{C}^n, \succeq, (p_j)_{j=1}^n, U]$ for $\succeq$, with all $p_j > 0$, and either $U: \alpha \mapsto \lambda \alpha^\rho$ for some $\lambda, \rho \in I\!R$ with $\lambda \rho > 0$, or $U: \alpha \mapsto \log \alpha$".

(ii) "The preference relation $\succeq$ is a continuous strongly monotonic weak order, it satisfies the sure-thing principle, and has constant relative risk aversion".

Proof

Suppose (i). Then, for any $\mu > 0$, $x \in \mathscr{C}^n$, for the expected utility EU, $EU(\mu x) = \mu^\rho EU(x)$ or $EU(\mu x) = \mu + EU(x)$. Constant relative risk aversion and all of (ii) follow straightforwardly from this. So we suppose (ii) and derive (i).

By strong monotonicity, every coordinate is "essential". By theorem 3 of Debreu [2], there exist continuous additive value functions $(V_j)_{j=1}^n$ for $\succeq$. By strong monotonicity, every V_j is strictly increasing. Define $V: \mathscr{C}^n \to I\!R$, $\phi: \mathscr{C} \to I\!R$, $W: \mathscr{C}^n \to I\!R$ by:

$$V: x \mapsto \sum V_j(x_j), \quad \phi: \alpha \mapsto V(\overline{\alpha}), \quad W: x \mapsto \phi^{-1} \circ V(x).$$

Then V and W represent $\succeq$; $W(\overline{\alpha}) = \alpha$; $[W(x) = \alpha \Rightarrow x \approx \overline{\alpha}]$, so W assigns to each x its "certainty equivalent". Also, $W(\mu x) = \mu W(x)$ for $\mu > 0$ (W is "linearly homogeneous", so V is "homothetic"). By Stehling ([8], theorem 2), or Eichhorn ([4], theorem 2.5.2), either:

$$V: x \mapsto \psi\left[\mu\left(\prod_{j=1}^n x_j^{p_j}\right)\right] \quad \begin{array}{l} \text{for a continuous strictly increasing } \psi, \\ \text{positive } \mu, \text{ and nonzero } p_1, \ldots, p_n \\ \text{that sum to one,} \end{array} \quad (3.4)$$

or:

$$V: x \mapsto \psi\left[\left(\sum_{j=1}^n \sigma_j x_j^\rho\right)^{1/\rho}\right] \quad \begin{array}{l} \text{for a continuous strictly increasing } \psi, \\ \text{positive } \sigma_1, \ldots, \sigma_n, \text{ and nonzero } \rho. \end{array} \quad (3.5)$$

In the case of (3.4), V is a strictly increasing transform of $x \mapsto \Pi x_j^{p_j}$, and so, by taking logarithms, of $x \mapsto \Sigma p_j \log(x_j)$. By strict increasingness of every V_j, every p_j is positive. So indeed we have an SEU model for $\succeq$, with $U: \alpha \mapsto \log \alpha$.

Next, suppose (3.5). First, assume $\rho > 0$. Then V is a strictly increasing transform of $\Sigma \sigma_j x_j^\rho$. We therefore have an SEU model for $\succeq$, with $p_j := \sigma_j / \Sigma_{i=1}^n \sigma_i$ for every j, and $U: \alpha \mapsto \alpha^\rho$, so $\lambda = 1$ in (i) above.

Finally, suppose (3.5), with $\rho < 0$. Then V is a strictly *decreasing* transform of $x \mapsto \Sigma \sigma_j x_j^\rho$, so a strictly increasing transform of $x \mapsto \Sigma \sigma_j(-x_j^\rho)$. We have an SEU model for $\succeq$, with $p_j := \sigma_j / \Sigma_{i=1}^n \sigma_i$ for every j, and $U: \alpha \mapsto -(\alpha^\rho)$, so in (i) above, $\lambda = -1$. □

From this, we derive:

THEOREM 3.5

Let $n \geqslant 3$. Let $\mathscr{C} = I\!R$. The following two statements are equivalent for the preference relation $\succeq$ on $\mathscr{C}^n$:

(i) "There exists an SEU model $[\mathscr{C}^n, \succeq, (p_j)_{j=1}^n, U]$ for $\succeq$, with all $p_j > 0$, and U identity, or $U: \alpha \mapsto \lambda e^{\rho \alpha}$ for some $\lambda, \rho \in I\!R$ with $\lambda \rho > 0$".

(ii) "The preference relation $\succeq$ is a continuous strongly monotonic weak order, it satisfies the sure-thing principle, and has constant absolute risk aversion".

Proof

Suppose (i). Then for any $\mu > 0$, $x \in \mathscr{C}^n$, the expected utility $EU(x)$ has $EU(x + \mu) = e^{\rho \mu} EU(x)$ or $EU(x + \mu) = \mu + EU(x)$. Constant absolute risk aversion and all of (ii) follow straightforwardly. So now we suppose (ii), and derive (i).

Define $L: I\!R_{++}^n \to I\!R^n$ by $L: (x_1, \ldots, x_n) \mapsto (\log(x_1), \ldots, \log(x_n))$, and define $\succeq'$ on $I\!R_{++}^n$ by $x \succeq' y$ iff $L(x) \succeq L(y)$. Then it follows straightforwardly that $\succeq'$ satisfies (ii) of theorem 3.4. We obtain, for all $x, y \in I\!R^n$:

$$x \succeq y \Leftrightarrow L^{-1}(x) \succeq' L^{-1}(y) \Leftrightarrow \sum p_j U(e^{x_j}) \geqslant \sum p_j U(e^{y_j}),$$

with U, p_j, and also λ, ρ as in (i) of theorem 3.4. This implies (i) of theorem 3.5. □

Most probably, the last two theorems also hold for any interval $\mathscr{C} \subset I\!R_{++}$, respectively, $\mathscr{C} \subset I\!R$, but we do not know of a reference where the analogue of Stehling's [8] theorem, needed to prove this, is available.

We have not considered the case of $n = 2$. One may replace $n \geqslant 2$ by $n = 2$ in theorems 3.3 to 3.5 if one adds the so-called "Thomsen condition" (see Krantz et al. [5], definition 6.3) in (iii) of theorem 3.3 and (ii) of theorems 3.4 and 3.5. For $n = 1$, the results are rather different; this concerns the case of decision making under certainty (and ordinal utility).

Uniqueness results in our theorems are standard, the utility function is always cardinal, and the subjective probabilities are uniquely determined.

4.   Conclusion

The approach by which the use of specified forms of subjective expected utility models is usually justified reflects the historical development of the theory of subjective expected utility maximization. Usually, in a first stage conditions are found, and justified, that are sufficient (and possibly necessary) for the existence of a subjective expected utility model without any further specification (for instance, the conditions in Savage [7]). Once this has been done, in a second stage behavioural conditions are found, and justified, which are sufficient (and possibly necessary) to obtain the desired specified form of the subjective expected utility model.

This paper has shown that a one-stage approach may lead to stronger results because assumptions on risk aversion, usually made in later stages, simplify the derivation in the first stage of the subjective expected utility model itself.

Acknowledgement

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