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The Rich Domain of Ambiguity Explored

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Abstract. Ellsberg and others suggested that decision under ambiguity is a rich empirical domain with many phenomena to be investigated beyond the Ellsberg urns. We provide a systematic empirical investigation of this richness by varying the uncertain events, the outcomes, and combinations of both. Although ambiguity aversion is prevailing, we also find systematic ambiguity seeking, confirming insensitivity. We find that ambiguity attitudes depend on the kind of uncertainty (the source) but not on the kind of outcomes. Ambiguity attitudes are closer to rationality (ambiguity neutrality) for natural uncertainties than for artificial Ellsberg urn uncertainties. This also appears from the reductions of monotonicity violations and of insensitivity. Ambiguity attitudes have predictive power across different outcomes and sources of uncertainty, with individual-specific components. Our rich domain serves well to test families of weighting functions for fitting ambiguity attitudes. We find that two-parameter families, capturing not only aversion but also insensitivity, are desirable for ambiguity even more than for risk. The Goldstein–Einhorn family performed best for ambiguity.

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1. Introduction

The first studies of ambiguity focused on the aversion found in the classical Ellsberg (1961) urns. Later studies revealed a richer picture. First of all, Trautmann and van de Kuilen's (2015) empirical review reports a fourfold pattern: prevailing ambiguity aversion for moderate to high likelihoods of gains and for unlikely losses, but prevailing ambiguity seeking for unlikely gains and most losses. Additionally, several authors have emphasized the importance of studying natural sources of uncertainty as occurring in real life, rather than studying the artificial sources almost exclusively studied in laboratory experiments. In the latter sources, ambiguity is created artificially by concealing information from subjects—for example, by concealing compositions of urns with colored balls (Ellsberg urns) or by giving only upper and lower bounds of probabilities to subjects. Camerer and Weber (1992, p. 361) wrote, "There are diminishing returns to studying urns!" Ellsberg (2011) himself also emphasized the richness of ambiguity and the importance of considering other phenomena, regarding both events and outcomes:

> . . . doesn't fully explain to me why nearly all later research has focused only on 'ambiguity aversion,' nor why most expositions have wrongly attributed the same *preoccupation* to me. . . . I happen to believe that this latter pattern [ambiguity seeking] will be much more frequent than the reverse in certain circumstances of payoffs and events other than the ones that were addressed explicitly in the QJE [*Quarterly Journal of Economics*] article and *almost exclusively investigated later*. Because these other circumstances . . . certainly deserving of much more experimental and theoretical investigation than it has received. (p. 226; italics added)

Other authors emphasizing the importance of studying natural events include Abdellaoui et al. (2005) and Heath and Tversky (1991, p. 6); Endnote 1 cites further papers studying natural events.

The domain of nonprobabilized uncertainties is rich just like the domain of nonmonetary commodities, with many kinds of informational and emotional configurations. One ambiguity attitude per subject for all nonprobabilized uncertainties is implausible, similar to one utility curve per subject for all nonmonetary commodities being implausible. To illustrate this point, Tversky and Fox (1995) showed that basketball fans are ambiguity seeking when the ambiguity concerns basketball, whereas they will continue to be ambiguity averse for most other sources. Although this finding is empirically unsurprising, it is useful as a first demonstration of the richness of ambiguity. Our paper follows up on the aforementioned findings and recommendations. We examine ambiguity attitudes toward various uncertain events ("sources"), various outcome domains, and combinations of both. Thus, we can compare source dependence with outcome dependence. This empirical comparison is important because it allows us to distinguish between the popular outcome-based ambiguity models, primarily the smooth model (Klibanoff et al. 2005), and event-based models (Ghirardato et al. 2004, Gilboa 1987, Schmeidler 1989, Tversky and Kahneman 1992). We further investigate how artificial ambiguity with information concealed from subjects differs from natural ambiguity and which parametric families best capture the richness of ambiguity. Numerous studies have investigated the performance of different parametric models for decision under risk. Focusing on nonexpected utility for risk, Online Appendix OA.1 cites 48 studies. Erev et al. (2010) reported a prediction competition between such models. This paper shows how such comparisons can be made for ambiguity. Because ambiguity is a richer domain than risk, parametric models for ambiguity also deserve extensive study.

2. Related Literature

It is well known that probability weighting for risk depends on the sign of outcomes (Tversky and Kahneman 1992) and, to some extent, on the size of outcomes (Etchart 2004, Fehr-Duda et al. 2010). For ambiguity, some theories model ambiguity attitudes through the utility of outcomes (Cerreia-Vioglio et al. 2015; Dobbs 1991; Klibanoff et al. 2005; Neilson 2010; Kahneman and Tversky 1975, pp. 30–33). Then, by definition, ambiguity attitudes depend on the outcomes considered. These theories are primarily normatively motivated. For normative applications in decision analysis, see Borgonovo and Marinacci (2015). Our purpose is, however, purely empirical.

We investigate outcome dependence by changing the nature of outcomes (money, waiting time, or life duration). This dependence has so far been investigated for risk but not yet for ambiguity. For risk, Rottenstreich and Hsee (2001) found that extreme outcomes can induce emotions that affect probability weighting. Abdellaoui et al. (2017) also found such dependence for monetary outcomes versus temporal outcomes, where time referred to waiting time with nothing to do (i.e., time lost), as relevant in transportation economics. Festjens et al. (2015) did not find such dependence. Kemel and Travers (2016) investigated decisions from experience (Hertwig et al. 2004), which can be considered to be intermediate between risk and ambiguity. Their results are similar to those of Abdellaoui and Kemel (2014). Armantier and Treich (2016) found that the probability weighting function can depend on the source that generates the probabilities even if all probabilities concerned are objective. Chew et al. (2012) similarly found a difference when the probabilities are generated by a digit of temperature in a known city versus an unknown city. Thus, there is some evidence of outcome-dependent and source-dependent probability weighting under risk. Yet it mostly occurs for emotional outcomes and sources, and it may not be very strong in general. In many applications, outcome-independent probability weighting will serve well as an approximation tractable enough to allow predictions for general emotion-neutral risks (Berns et al. 2007).

Some studies considered natural sources of uncertainty[1] and have demonstrated that ambiguity attitudes depend on the source. We are not aware of studies that have investigated the dependence of ambiguity attitudes on kinds of outcomes or on combinations of outcomes and events, or that have tested parametric families for ambiguity. We consider three of the most important outcomes: (a) money, which is the most studied outcome in economics; (b) delayed time of receiving an outcome, widely investigated in the literature on discounting; and (c) life duration, the most important outcome in the health domain. McFadden (2010) suggested that studying ambiguity with time as outcome, as in (b), is important. Several studies considered this topic (see Kemel and Paraschiv 2013 and references therein). The only study that considered both variations in outcomes and events under ambiguity is Eliaz and Ortoleva (2016). They investigated effects of correlations on ambiguity attitudes. Calibrating ambiguity attitudes or their dependence on outcomes or events was not the purpose of their study.[2]

3. Theory on Ambiguity Attitudes: The Source Method and α-Maxmin Expected Utility

Gilboa and Marinacci (2013) and Machina and Siniscalchi (2014) reviewed the theoretical and normative literature on ambiguity aversion. Our descriptive analysis of ambiguity is based on biseparable utility, which is a convenient point of departure for many popular ambiguity models (Ghirardato and Marinacci 2001). *Biseparable utility* evaluates a *binary prospect* $\gamma_E\beta$, yielding outcome γ if uncertain event E occurs and outcome β otherwise, with γ preferred to β by $W(E)U(\gamma) + (1 - W(E))U(\beta)$. Here, U is the usual *utility function* and W is a nonadditive *event weighting function* ($W(\varnothing) = 0$, $A \supset B \Rightarrow W(A) \geq W(B)$, and $W(S) = 1$ for the universal event S). We only consider gains in this paper and therefore do not need to discuss sign dependence. Biseparable utility comprises multiple priors, α-maxmin, prospect theory for gains (and for losses), and Choquet expected utility (Wakker 2010, Section 10.6). Thus, our results pertain to all these theories. We use the source method (Abdellaoui et al. 2011), a tractable specification of biseparable utility based on Chew and Sagi's (2008) axioms. In Kothiyal et al. (2014), the source method predicted ambiguous choices better than a number of popular alternative models. We will also discuss our results from the perspective of the popular α-maxmin model (Ghirardato et al. 2004).

In the source method, subjective probabilities are specified for each source of uncertainty. They are called *ambiguity neutral*, or *a-neutral*, and they are transformed into ambiguity decision weights. Although it was long believed, based on Ellsberg's paradoxes, that probabilities cannot be used to model ambiguity, Chew and Sagi (2008) showed that they can still be used by allowing decision attitudes to depend on the source of uncertainty. Thus, an a-neutral probability 0.5 for an ambiguous Ellsberg urn is transformed more pessimistically than an objective probability 0.5, implying ambiguity aversion as in the Ellsberg paradox. *Sources* of uncertainty are groups of events generated by the same uncertainty mechanism. This concept was proposed by Heath and Tversky (1991) and analyzed systematically by Tversky and Fox (1995). The three sources of uncertainty that we consider in our experiment concern (1) which of 10 possible colors does a ball drawn from an Ellsberg urn have, (2) which of 10 possible districts does a child from India come from, and (3) which of 10 possible viruses caused a disease.

We use Dimmock et al.'s (2016) simplified implementation of the source method. These authors deliberately minimized the number of measurements and the experimental time per subject so as to demonstrate the tractability of their method. We use more detailed and thorough measurements and more time per subject so as to obtain better reliability and validity.

This section explains how we measured the ambiguity indexes for the Ellsberg urn. We apply the same method for the other two sources of uncertainty. The basic setting is an urn that contains 100 colored balls. Each ball has been painted in 1 of 10 colors. Suppose there is one winning color—say, red. One ball is drawn randomly from the urn. If the ball is red, subjects receive a good outcome (say, €500). Otherwise, they receive a bad outcome (i.e., nothing). Subjects thus consider *gambles* $\gamma_E\beta$ on events E, yielding a good outcome γ if event E happens and a bad outcome β otherwise. We considered events E_j of j winning colors for $j = 1$, 3, 5, 7, and 9, where higher j's give more favorable events because their likelihoods are higher. Subjects had the same information about all colors, and they had no reason to consider any one more likely than any other. We therefore made the common assumption that subjects have no color preference and thus satisfy Chew and Sagi's (2008) exchangeability axiom. As Chew and Sagi showed, an ambiguity-neutral (Bayesian) decision maker would therefore assign subjective probability $j/10$ to event E_j. We call $j/10$ the *ambiguity-neutral* (*a-neutral*) *probability* of event E_j. For each event E_j, we elicited the *matching probability* $m(j/10)$, being such that a subject considered gaining γ with objective probability $m(j/10)$ to be equivalent to gaining γ under event E_j. The function $m(\cdot)$ depends on the source of uncertainty, which can be expressed by adding a subscript: $m_{\mathrm{So}}(\cdot)$.

For an ambiguity-neutral decision maker, we have $m(j/10) = j/10$ for all j. For general decision makers and each event E_j, the difference

$$AA_j = \frac{j}{10} - m\left(\frac{j}{10}\right), \tag{1}$$

which is the difference between the a-neutral probability and the matching probability, serves as an *event-dependent ambiguity aversion index*. Ambiguity-averse subjects dislike the ambiguity comprised in E_j, and a small objective probability $m(j/10) < j/10$ will then be equivalent to E_j, implying Equation (2). We have

$$\frac{j}{10} - m\left(\frac{j}{10}\right) > 0: \quad \text{ambiguity aversion for } E_j; \tag{2}$$

$$\frac{j}{10} - m\left(\frac{j}{10}\right) = 0: \quad \text{ambiguity neutrality for } E_j; \tag{3}$$

$$\frac{j}{10} - m\left(\frac{j}{10}\right) < 0: \quad \text{ambiguity seeking for } E_j. \tag{4}$$

Thus, the matching probabilities $m(j/10)$ provide an easy tool to measure ambiguity attitudes. Dimmock et al. (2016, Theorem 3.1) gave a theoretical justification, showing that matching probabilities easily and completely capture ambiguity attitudes for biseparable utility. Knowledge of the risk attitude and of matching probabilities indeed fully capture preferences over binary gambles.

Dimmock et al. (2016) derived global indexes of ambiguity attitudes as follows. As an intermediate step of recoding data (and not of statistical estimation), we determine the best-fitting (by quadratic distance) line

$$p_{\text{a-neutral}} \rightarrow c + s \times p_{\text{a-neutral}} \tag{5}$$

for the six data points $(j/10, m(j/10))$ in which $j = 1$, 3, 5 (twice), 7, and 9. Here, $s \geq 0$ and the fitted values are truncated at 0 and 1; that is, they should not be negative or exceed 1. Although most readers use best-fitting lines for statistical purposes, our purpose is not statistical at this stage. This line only serves as an intermediate step in a mathematical and deterministic calculation of the indexes. No probabilistic model has been specified here and no statistical claims have been made at this stage.

It is natural that ambiguity aversion is higher as the values $m(j/10)$ are lower, analogously to Schmeidler's (1989, pp. 572, 574) index of ambiguity aversion (defined in Equation (11)). We thus define the following index (a linear rescaling of the area above the line in Equation (5)):

$$b = 1 - s - 2c \text{ is the } \textit{index of ambiguity aversion}. \quad (6)$$

This component is motivational, reflecting an overall liking or disliking of ambiguity. Under expected utility (ambiguity neutrality) $c = 0$ and $s = 1$, and the index has value 0. Positive values indicate ambiguity aversion, with 1 being the maximum value, whereas negative values indicate ambiguity seeking, with −1 being the minimum value.

Furthermore,

$$a = 1 - s \text{ is the } \textit{index of a(mbiguity-generated likelihood)-insensitivity}. \quad (7)$$

This index reflects the shallowness of $m(\cdot)$ in the middle region and, hence, insensitivity toward gradual changes in likelihood of the E_j events. Insensitivity is most naturally interpreted as a cognitive component of ambiguity, reflecting general (lack of) understanding of likelihood. Under expected utility we have $a = 0$, reflecting optimal sensitivity. The index a is usually positive, reflecting lack of sensitivity. Dimmock et al. (2016) gave further explanations and a theoretical background. The two indexes reflect distances from ambiguity neutrality. They satisfy the desirable property of utility independence of Baucells and Borgonovo (2014).

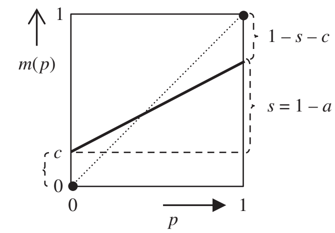
The follow-up paper Baillon et al. (2017) introduced generalized indexes that can also be applied to events that do not have Ellsberg-type symmetries, and it discussed the similarities and differences with other existing indexes. In particular, the preceding analysis can be reinterpreted using the α-maxmin model. The level of perceived ambiguity in this model is identical to our a, and the ambiguity aversion index α of that model is our aversion index b per unit of perceived ambiguity ($\alpha \approx b/a$). Hence, the two indexes of the two theories contain the same information. Dimmock et al. (2015) used the alternative interpretation based on the α-maxmin model.

We also use parametric families to fit the data. We estimate how the matching probabilities are a function of the a-neutral probabilities. The parametric families that we use have commonly been used for probability weighting for decision under risk, capturing risk attitudes. For risk, as for ambiguity, aversion ("pessimism") and insensitivity are central, which is why these families can be expected to be suitable for analyzing ambiguity in the same way as they are for risk. We use them here for fitting the matching probabilities $m(j/10)$ and consider the following three families, the first popularized by Chateauneuf et al. (2007).

1. *Neo-additive* (see Figure 1):

$$\begin{gathered} m(0) = 0; \quad m(1) = 1; \\ m(p) = c + sp \quad \text{for } 0 < p < 1;\ s \geq 0; \\ m(\cdot) \text{ is truncated at the values 0 and 1.} \end{gathered} \quad (8)$$

Figure 1. The Neo-Additive Family



Indexes of a-insensitivity and ambiguity aversion are as defined in Equations (6) and (7).

2. *Goldstein and Einhorn* (1987):

$$m(p) = \frac{\beta p^{\alpha}}{\beta p^{\alpha} + (1-p)^{\alpha}}; \quad \alpha \geq 0,\ \beta \geq 0. \quad (9)$$

Here, α is often used as an (anti-)index of a-insensitivity, similar to our a, and β is an (anti-)index of ambiguity aversion, similar to our b.

3. *Prelec* (1998) *two-parameter*:

$$m(p) = (\exp(-(-\ln(p))^{\alpha}))^{\beta}; \quad \alpha \geq 0,\ \beta \geq 0. \quad (10)$$

Here, α is often used as an (anti-)index of a-insensitivity, and β is an index of ambiguity aversion. Online Appendix OA.7 also reports results for Prelec's (1998) and Tversky and Kahneman's (1992) one-parameter families, not defined here.

4. Experimental Design

4.1. The Basic Treatment

We considered five treatments—that is, five combinations of sources and outcomes—displayed in Table 1. The online appendix contains the exact wordings of the instructions for subjects. We partially randomized the order of presentation of the treatments by using two different orders: week, basic, year, health, and kid; and the partly reversed order: health, year, basic, week, and kid. The kid treatment is always the last because it was designed to arouse specific emotions and could distort the other decisions.

This subsection presents the first treatment (i.e., the basic treatment), which concerns a standard Ellsberg experiment. Two urns both contained 100 balls with possibly up to 10 different colors: yellow, orange, red, dark pink, light pink, purple, dark blue, light blue, light

Table 1. The Five Treatments

Treatment	Source of uncertainty	Outcome
Basic	Ellsberg urn	Money
Week	Ellsberg urn	Waiting time (weeks)
Year	Ellsberg urn	Waiting time (years)
Kid	Districts	Money
Health	Viruses	Life duration

Figure 2. Screenshot of Choosing the Winning Colors

In this particular question, you can bet on three colors. If the ball drawn is any of these three colors, you win the bet.

Select the **three** colors you want to bet on.

☑ yellow ☑ orange ☑ red ☐ dark-pink ☐ light-pink ☐ purple ☐ dark-blue ☐ light-blue ☐ light-green ☐ dark-green

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green, and dark green. The composition of balls was known in urn K but unknown in urn U. The composition of urn U had been prepared in advance by an outside party (a secretary). In other words, the experimenters did not know the composition of urn U during the experiment. Subjects were informed about the preparation of urn U so that they knew that the experimenters could not influence its composition during the experiment.

For each $j = 1$, 3, or 5, subjects first chose which j out of 10 colors were the winning colors, determining the winning event E_j for both urns (see Figure 2). Subjects next chose from which urn, U or K, a ball is randomly drawn. A choice list was used to determine the urn K that yields indifference. Figure 3 gives an example.[3] The three winning colors were yellow, orange, and red. Urn U is on the right side, and the 11 K urns are on the left, 1 in each row, with the number of winning balls specified. This number was different in different choice situations (rows). Subjects chose between K and U in each row, marking their preference in the middle columns. As usual, no indifference was allowed. If the color of the ball drawn was a winning color, the subject received a good outcome (€500); otherwise, the subject received a bad outcome (€0). If the implementation of the real choice situation at the end involved urn K (i.e., if the subject had chosen urn K), then this urn was prepared with the proper composition by the experimenters in front of the participants of that session.[4]

For each E_j, we elicited choices for all 101 compositions of winning balls in urn K using the incentive compatible implementation of refined choice lists introduced by Abdellaoui et al. (2011), which will be explained next. Subjects preferred urn U for low numbers of winning balls in K, and urn K for high numbers. They switched preferences somewhere in between these 101 choices. We measured this switching point in two steps, as follows. A first choice list (see Figure 3) included 11 choices between urn K and urn U, with $0, 10, \ldots, 100$ balls of the winning color(s) in urn K. After having made their choices as in Figure 3, subjects were shown a more refined choice list. The second choice list (see Figure 4) was refined between the two values in the first choice list where the preference switch had happened. In Figure 3, switching happened between 30 and 40, so that the next choice list in Figure 4 included the choices for $31, 32, \ldots,$ 39 balls of winning color(s) in urn K. Here, switching happened between 35 and 36. This procedure allowed us to infer the subject's choices for all 101 compositions of urn K.

Figure 3. (Color online) Screenshot of Choice List: First Step

You have chosen the colors **yellow, orange, and red**, which means that you win the bet if a **yellow, orange, or red** ball is drawn.

Look at the choice list below. You can now decide which bag (Bag K or Bag U) to draw the ball from.

Number of Balls in Bag K		K	U	Number of Balls in Bag U	
€500: yellow, orange, and red	€0: the other colors			€500: yellow, orange, and red	€0: the other colors
0	100	○ Bag K	◉ Bag U	**unknown**	100 - unknown
10	90	○ Bag K	◉ Bag U		
20	80	○ Bag K	◉ Bag U		
30	70	○ Bag K	◉ Bag U		
40	60	◉ Bag K	○ Bag U		
50	50	◉ Bag K	○ Bag U		
60	40	◉ Bag K	○ Bag U		
70	30	◉ Bag K	○ Bag U		
80	20	◉ Bag K	○ Bag U		
90	10	◉ Bag K	○ Bag U		
100	0	◉ Bag K	○ Bag U		

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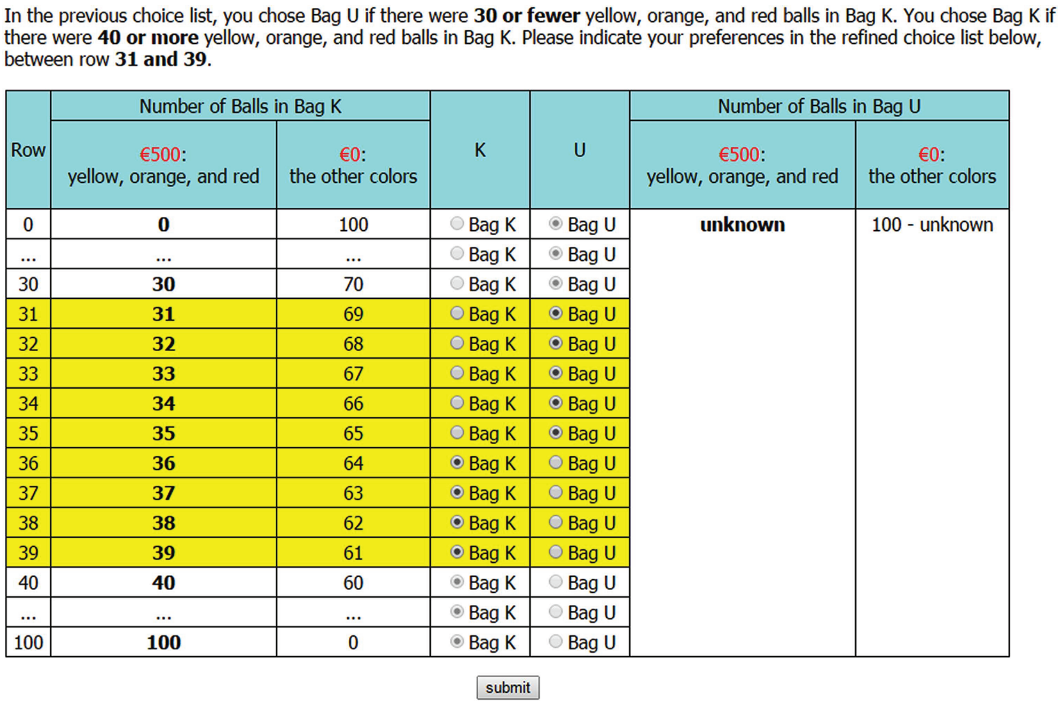
Figure 4. (Color online) Screenshot of Choice List: Second Step



In the previous choice list, you chose Bag U if there were **30 or fewer** yellow, orange, and red balls in Bag K. You chose Bag K if there were **40 or more** yellow, orange, and red balls in Bag K. Please indicate your preferences in the refined choice list below, between row **31 and 39**.

Row	Number of Balls in Bag K		K	U	Number of Balls in Bag U	
	€500: yellow, orange, and red	€0: the other colors			€500: yellow, orange, and red	€0: the other colors
0	**0**	100	○ Bag K	◉ Bag U	**unknown**	100 - unknown
...	...	...	○ Bag K	◉ Bag U		
30	**30**	70	○ Bag K	◉ Bag U		
31	**31**	69	○ Bag K	◉ Bag U		
32	**32**	68	○ Bag K	◉ Bag U		
33	**33**	67	○ Bag K	◉ Bag U		
34	**34**	66	○ Bag K	◉ Bag U		
35	**35**	65	○ Bag K	◉ Bag U		
36	**36**	64	◉ Bag K	○ Bag U		
37	**37**	63	◉ Bag K	○ Bag U		
38	**38**	62	◉ Bag K	○ Bag U		
39	**39**	61	◉ Bag K	○ Bag U		
40	**40**	60	◉ Bag K	○ Bag U		
...	...	...	◉ Bag K	○ Bag U		
100	**100**	0	◉ Bag K	○ Bag U		

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If for j winning balls in U preferences switched between i and $i+1$ winning balls in K, we estimated the matching probability $m(j/10)$ to be $(i+1/2)/100$. If there were no switches, then $m(j/10)$ was 0 or 1, as the case may be. The program did not allow for multiple switches. Following Abdellaoui et al. (2011), we randomly selected 1 from the 101 compositions of urn K, and not only from the choices actually asked in the two choice lists. This ensured incentive compatibility.

For each determination of $m(j/10)$ as just described and $j = 1, 3, 5$, we immediately after considered the same set of j colors, except now they were the losing colors, whereas the other $10-j$ colors now were the winning colors. Hence, we asked six questions (with different winning colors) in each treatment so that we could determine six values, $m(j/10)$ and $m(1-j/10)$, $j = 1, 3, 5$.[5]

4.2. Alternative Treatments

4.2.1. The Week Treatment. In the week treatment, the second one, we changed the outcome into waiting time (for receiving an outcome) instead of money. Subjects still received €250 with certainty. This amount was chosen to achieve the same level of average payoff as in the basic treatment, as is common in experiments. But now the uncertainty concerned the time when subjects received the €250. The good outcome meant receiving the money immediately, whereas the bad outcome meant receiving it eight weeks later. Interactions between money, time, and uncertainty, as in the magnitude effect (Baucells and Heukamp 2012), played no role in our design because, first, money was kept constant and, second, all that matters was that there was a good and a bad outcome.

4.2.2. The Year Treatment. The third treatment, the year treatment, was similar to the week treatment, the only difference being that the money to be won with certainty amounted to €5,000 and that the time of receipt was either immediately or in 10 years. Choices in this year treatment were hypothetical—serving to test the hypothetical bias—and subjects received an immediate flat payment of €250 if this treatment was selected for implementation. In every other respect, the week and year treatments were the same as the basic treatment, using the same Ellsberg urns and adopting the same method to measure matching probabilities $m(j/10)$ and $m(1-j/10)$, $j = 1, 3, 5$.

4.2.3. The Kid Treatment. In the fourth treatment, the kid treatment, we did not change the outcomes (€500 or €0) relative to the basic treatment, but instead we changed the source of uncertainty. The source involved a charitable program in rural India, paying for children's school education. We showed our subjects a photo[6] of one of the children whose lives had been transformed by this charitable program.[7]

The child came from 1 of 100 villages that were distributed over 10 possible districts: Ludhiana, Sangrur, Amritsar, Kaithal, Sonipat, Jodhpur, Pali, Udham Singh Nagar, Bulandshahr, and Shahjehanpur. Subjects could now gamble on the district of the child's village. They bet on the winning districts (instead of colors). The ambiguous option in this treatment is called option C (for "charity") and the risky option is called urn K (for "known").

Our subjects could not be expected to have any geographic knowledge of the concerned villages or districts, or their sizes. Thus the 10 districts represented equally likely events to our subjects in the same way as the 10 colors in the Ellsberg urn were equally likely events. The 100 villages are analogous to the 100 balls in the Ellsberg urn; neither of them is outcome relevant beyond district/color. Both the photo and the charitable context (related to school education) can be expected to arouse positive emotions,[8] which may offset the negative emotions generated by us by concealing information about the districts from our subjects. Hence, this treatment could be called the feel-good treatment. Matching probabilities were measured using the same procedure with a known urn K as before. Now each district was coupled with a color, so that gambling on three districts corresponded to gambling on three colors in the known urn, and so on.

The uncertainty in this treatment is less artificial than in the preceding treatments in the sense that the uncertainty refers to real, natural events rather than to drawing balls from urns only for the purpose of the experiment. Yet they are still artificial in the sense that information is deliberately kept secret from subjects. Hence, the ambiguity here is intermediate between artificial and natural.

4.2.4. The Health Treatment. The fifth and final treatment was a health treatment, which deviated more from the basic treatment than the other treatments. We now changed both the outcomes and the source of uncertainty. This treatment was again hypothetical, and subjects received an immediate flat payment of €250 if it was selected for implementation. We used a virus story for the source of uncertainty (see Figure A.1). The subjects were asked to imagine that they had been diagnosed with a particular disease and that they would have to receive medical treatment. They were told that there were 10 possible mutually exclusive viruses (numbered from 1 to 10) causing the exact same disease. There was no way to diagnose which virus caused the disease, but the disease could only be cured if the real virus was treated. In the case of recovery (disease cured), the subjects would live 50 years longer in good health, and otherwise, they would live only one year longer in good health. In other words, now the outcome was life duration. Specifying a particular life duration may seem unrealistic, but it is still widely used in the health domain for various reasons (Gold et al. 1996). Therefore, its study is important.

Subjects were asked to choose between treatment K and treatment U. Treatment K would involve a broad-spectrum antiviral supplement with a known success rate (given in percent). Treatment U was new and would use specific supplements (numbered from 1 to 10), which would only be effective against the virus with the corresponding number. The disease could be cured only if the right supplement for the real virus was chosen. Because the subjects were told that there was no way to diagnose which virus was causing the disease, the 10 viruses were equally likely to them in the same way as the Ellsberg colors or the districts were. Event E_j now meant that only j supplements could be provided. To measure matching probabilities $m(j/10)$, we did not use a known urn, but treatment K with success rates specified for 0%, 1%, ..., 100%.

Because this fifth health treatment was hypothetical, subjects did not choose the j supplements provided as they chose the j colors in the basic treatment. Instead, the first j supplements were offered in treatment U. Neither suspicion nor illusion of control, the common confounds in Ellsberg experiments, played a role here.

The uncertainty in this source is not artificial in the sense that it does not result from an experimenter deliberately concealing information from subjects, but it is caused by extraneous lack of information, as is common in applications. In this sense, this treatment is the most natural one in this study.

4.3. Further Experimental Details

4.3.1. Subjects. We recruited $N = 66$ subjects (73% male, 27% female), bachelor's and master's students from various fields, online from the ESE-EconLab website of the Erasmus School of Economics.

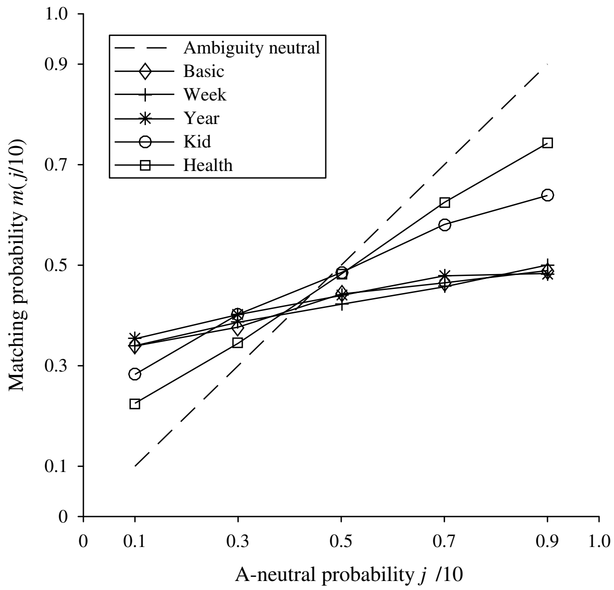
4.3.2. Procedure. The experiment was conducted at the experimental laboratory of Erasmus University Rotterdam. There were three sessions, all on the same day.[9] The sessions lasted one and a half to two hours.

4.3.3. Incentives. Subjects received a show-up fee of €5. One randomly selected subject in each of the three sessions received an additional payment. We first randomly selected which of the five treatments would be implemented. Two treatments were hypothetical, for which a fixed payment of €250 was given. For the other three, one randomly selected choice was implemented. If it was an uncertain prospect, then the relevant uncertain information was revealed to the subject. Total average earnings were €16.36. All random selections were noncomputerized and verifiable to the subjects, implemented by drawing balls from urns.

5. Results

As the different orders of treatments mostly gave no differences (see Online Appendix OA.2), we pooled the data. In short, our findings regarding the indexes are as follows. Principal component analyses (in Online Appendices OA.5 and OA.6) show that our two indexes capture most of the variance of the ambiguity attitudes. Changing the outcomes does not affect the ambiguity attitudes, but changing the sources of uncertainty does. We find lower aversion and better sensitivity in the kid

Figure 5. Mean Matching Probability



and health treatments. Analyses using the parametric families of weighting functions confirm the aforementioned findings. The Goldstein and Einhorn family fits the data best. The indexes of all families are strongly correlated across different treatments, showing predictability across sources of uncertainty and person-specific components. We next give details.

5.1. Indexes b and a, and Outcome Dependency vs. Event Dependency

Figure 5 plots the mean matching probabilities $m(j/10)$, $j = 1, 3, 5, 7, 9$, and it displays the main phenomena, which will later be confirmed by statistical tests. The curves are somewhat below 0.5 on average, meaning that there is more ambiguity aversion than ambiguity seeking. For low likelihoods, there is prevailing ambiguity seeking, in agreement with a-insensitivity. The curves are almost linear in the interior, suggesting that neo-additive functions fit the data well, in agreement with common findings (Baucells and Villasís 2015, Trautmann and van de Kuilen 2015).

When we compare between treatments, we find that the curves of the three Ellsberg treatments (basic, week, and year) are very similar. Outcomes therefore do not affect ambiguity attitudes. However, in the kid and health treatments, changes in the source of uncertainty do affect ambiguity attitudes. In particular, sensitivity becomes better as the ambiguity becomes more natural.

Table 2 analyses ambiguity attitudes per event (Equations (2)–(4)), presenting the median event-dependent ambiguity aversion index (Equation (1)) per event and treatment. For the basic, week, year, and kid treatments, it shows ambiguity seeking for the unlikely events E_1 and E_3 and ambiguity aversion for all other events, except for ambiguity neutrality[10] for E_5 in the kid treatment. The index is negative and close to 0 for the health treatment for all the events, with E_1, E_3, and E_9 displaying ambiguity seeking.

Table 3 presents estimations of the indexes assuming that subjects are homogeneous and then minimizing overall linear least squares.[11] Standard errors are corrected for clustering at the subject level. There is some ambiguity aversion, but it is close to neutral (0), and it is not significant for the kid and health treatment. A-insensitivity is strong. Changes to outcomes do not affect the indexes, which are the same for the basic, week, and year treatments ($p > 0.52$ and $p > 0.56$ for b and a). Changing the source of uncertainty from the basic to the kid treatment gives lower ambiguity aversion (because of prior expectation, one-sided test: $p < 0.01$) and much better sensitivity ($p < 0.001$). The health treatment shows yet more sensitivity than the kid treatment ($p < 0.01$), but aversion is not significantly different ($p = 0.84$).

Table 2. Ambiguity Attitudes per Event

A-neutral probability $j/10$	Median event-dependent ambiguity aversion index AA_j (percentage of subjects with the majority ambiguity attitude)				
	Basic	Week	Year	Kid	Health
0.1	-0.245^{***} (87.88%)	-0.245^{***} (89.39%)	-0.280^{***} (84.85%)	-0.085^{***} (84.85%)	-0.005^{***} (81.82%)
0.3	-0.065^{***} (80.30%)	-0.100^{***} (74.24%)	-0.105^{***} (71.21%)	-0.050^{***} (80.30%)	-0.005^{***} (74.24%)
0.5	0.035^{***} (77.27%)	0.010^{***} (75.76%)	0.005^{***} (66.67%)	0.005 (53.03%)	−0.005 (66.67%)
0.7	0.230^{***} (95.45%)	0.205^{***} (96.97%)	0.205^{***} (89.39%)	0.100^{***} (71.21%)	−0.005 (56.06%)
0.9	0.430^{***} (96.97%)	0.405^{***} (93.94%)	0.410^{***} (98.48%)	0.205^{***} (75.76%)	-0.005^{**} (51.52%)

$^{*}p \leq 0.10$; $^{**}p \leq 0.05$; $^{***}p \leq 0.01$.

Table 3. Overall Ambiguity Attitudes Across Treatments

	Basic	Week	Year	Kid	Health
Ambiguity aversion index b	0.15***	0.16***	0.13***	0.04	0.03
A-insensitivity index a	0.81***	0.80***	0.83***	0.55***	0.34***

$^{*}p \leq 0.10$; $^{**}p \leq 0.05$; $^{***}p \leq 0.01$.

5.2. Individual Consistency

Table 4 reports Spearman's rank correlation coefficients of each of the indexes across all five treatments, where the upper right triangle shows the ambiguity aversion index b and the lower left triangle shows the a-insensitivity index a. Correlations among the basic, week, and year treatments are highly significant. The kid treatment is dissimilar, and the health treatment even more so. For the a-insensitivity index a, the health treatment does not even have a significant correlation with the basic, week, and year treatments. These findings confirm, for both indexes, that cases with the same source of uncertainty but different outcomes are more similar than cases with the same outcomes but different sources of uncertainty. Here, the kid treatment is intermediate between Ellsberg uncertainty and natural uncertainty.

5.3. Discussion of Rationality

Because of monotonicity, $m(j/10)$ should be increasing in j. We find much insensitivity, with $m(\cdot)$ only weakly increasing with a shallow slope. Because of this, and because of the randomness that is common in decision experiments, there are many violations of monotonicity at the individual level. We test monotonicity in all possible cases. The second row in Table 5 gives the percentages of violations for the five treatments. These relatively high percentages—higher than commonly found for decision under risk—confirm that there is more insensitivity (lack of understanding) under ambiguity. They also show that choices are most rational in the health treatment, second-most in the kid treatment, and they are least rational, about equally, in the remaining three treatments.

Table 5. Violations of Monotonicity and Correlations Between Indexes

	Basic	Week	Year	Kid	Health
Violations of monotonicity (%)	25	28	27	19	14
Correlations between indexes b and a	0.45***	0.39***	0.35***	0.29**	0.15

$^{*}p \leq 0.10$; $^{**}p \leq 0.05$; $^{***}p \leq 0.01$.

Although ambiguity aversion and a-insensitivity are conceptually distinct, they may well be empirically correlated. A positive correlation is natural because both indexes concern deviations from Bayesianism and, according to many, deviations from rationality. The bottom row in Table 5 gives the Spearman's rank correlations of the two indexes for each of the five treatments. The correlations are all significantly positive except for the health treatment (where there is less irrationality).

5.4. Parametric Fittings

We use least squares data fitting, which equals the maximum log-likelihood method when assuming Fechner error. We also did fitting at the individual level, reported in Online Appendix OA.7. Those results confirm all results reported here. One-parameter families performed poorly in all respects and are reported in Online Appendix OA.11. Table 6 shows that for each treatment the ordering of goodness of fit of parametric families by Akaike's information criterion (AIC) is (1) neo-additive, (2) Goldstein and Einhorn, and (3) Prelec two-parameter. The AIC corrects for the number of parameters used, but still, the two-parameter families are superior to the one-parameter families, as reported in Online Appendix OA.11. It is clearly important to consider both the aversion and the insensitivity component when studying ambiguity, and focusing on one (Prelec one-parameter considers only insensitivity) or combining the two (Tversky and Kahneman 1992) loses too much explanatory power. Other than this, the ordering of parametric fit found is different than for risk (Balcombe and Fraser 2015). The reason is that insensitivity plays a more central role for ambiguity than for risk. Hence the neo-additive

Table 4. Correlations of Individual Ambiguity Attitudes Across Treatments

		Ambiguity aversion index b				
		Basic	Week	Year	Kid	Health
A-insensitivity index a	Basic		0.50***	0.64***	0.24*	0.21*
	Week	0.56***		0.53***	0.27**	0.18
	Year	0.57***	0.40***		0.47***	0.32***
	Kid	0.32***	0.24*	0.11		0.16
	Health	0.06	−0.04	−0.03	0.36***	

$^{*}p \leq 0.10$; $^{**}p \leq 0.05$; $^{***}p \leq 0.01$.

Table 6. Fit of Parametric Families: AIC

Parametric family	Basic	Week	Year	Kid	Health
Neo-additive	−281.1	−148.7	−201.2	−250.2	−167.2
Goldstein and Einhorn	−280.1	−148.6	−200.6	−248.6	−164.8
Prelec two-parameter	−277.9	−148.0	−199.0	−243.8	−164.1

Table 7. Fitted Parameters (Significance Level Given by Comparison with Basic Treatment)

Parametric family	Parameters	Basic	Week	Year	Kid	Health
Neo-additive	c	0.33	0.32	0.35	0.26**	0.15***
	s	0.19	0.20	0.17	0.45***	0.66***
Goldstein and Einhorn	$\beta^{\downarrow}$	0.74	0.73	0.76	0.92***	0.93***
	$\alpha^{\downarrow}$	0.15	0.15	0.13	0.35***	0.55***
Prelec two-parameter	β	0.91	0.93	0.89	0.86	0.92
	$\alpha^{\downarrow}$	0.14	0.14	0.12	0.35***	0.56***

Note. A down arrow ($^{\downarrow}$) denotes an anti-index.
*$p \leq 0.10$; **$p \leq 0.05$; ***$p \leq 0.01$.

family, which can readily handle extreme degrees of insensitivity, fares best. The fact that Goldstein and Einhorn perform better to some extent than Prelec's two-parameter family may be because the former better separates the two parameters. In Prelec's family, the insensitivity parameter α overlaps partly with the aversion parameter β, also capturing some aversion.

Table 7 reports the fitted parameters of the parametric families (all significant at the 1% level).[12] Comparing across treatments, changes of outcomes do not affect the parameters, where the week and year treatments are the same as the basic treatment. Changing the source of uncertainty in the kid and health treatments gives better sensitivity and lower ambiguity aversion judging by the parameters, except in the Prelec two-parameter family where the ambiguity aversion parameter b is constant across all treatments. The neo-additive family gives lower parameters c and s for the health treatment than for the kid treatment (both $p < 0.01$). The Goldstein and Einhorn two-parameter family and the Prelec two-parameter family both give the same β ($p = 0.90$ and $p = 0.29$, respectively) and a higher α (one-sided test: $p < 0.001$ and $p < 0.01$, respectively) for the health treatment.

5.5. Discussion of Experimental Details

To control for suspicion (the experimenters rigging urns/districts), subjects could choose the colors/districts to gamble on for the basic, week, and year treatments with the unknown Ellsberg urn and for the kid treatment with unknown regions. Immediately after having gambled on an event, we had the subjects gamble on its complement. This further made clear to subjects that we had no interest in rigging urns or districts.

We grouped events and their complements together to make likelihoods clearer to subjects and thus obtain replies of higher quality. This also allowed us to directly measure Schmeidler's (1989) indexes of ambiguity aversion, as follows. In Schmeidler (1989), as a consequence of the expected utility assumed for risk, the matching probabilities $m(j/10)$ are the weights of the events E_j. Schmeidler (1989, pp. 572, 574) and Dow and Werlang (1992) proposed

$$1 - m\left(\frac{j}{10}\right) - m\left(1 - \frac{j}{10}\right) \tag{11}$$

as indexes of ambiguity aversion in terms of the weighting function. The indexes are the sum of the event-dependent ambiguity aversion indexes (Equation (1)) of event E_j and its complement E_{10-j}, and they have been widely used since. Our index b of ambiguity aversion is an aggregate of these indexes for the pairs (E_1, E_9), (E_3, E_7), and (E_5, E_5^c). By using matching probabilities instead of Schmeidler's weighting function, we make the indexes directly observable, and, unlike Schmeidler (1989), do not need to assume expected utility for risk, as shown by Baillon et al. (2017).

In one respect, the health treatment is not realistic, which forced us to resort to hypothetical choice: we should have exchangeable symmetric uncertainties. This is needed for direct comparability with the Ellsberg urn where such symmetry is central. Such symmetries are virtually absent from practice, and therefore it is virtually impossible to come up with a realistic example of this kind with real incentives. This difficulty can be held against the representativeness of the Ellsberg urns for applications. It is more interesting to study natural sources of uncertainty. For this reason, Baillon et al. (2017) introduced new indexes of ambiguity attitudes that can handle natural events without symmetries.

The year treatment in our experiment was also hypothetical, even though there are many reasons to prefer real incentives to hypothetical choice. One of the aims of this treatment was to test for the hypothetical bias. The absence of a difference between the hypothetical year treatment and the incentivized week treatment suggests that there is no hypothetical bias in our design. The good quality of the results in the health treatment—better than in the other treatments—and our apparently well-motivated subjects there further suggest that we have no hypothetical bias.

6. General Discussion

We first discuss the basic treatment with the classical Ellsberg urns (10 colors) and monetary outcomes. Here, we find the usual prevailing ambiguity aversion as can be seen from our b-index. In particular, 77% of the subjects exhibited ambiguity aversion for the ambiguous 50-50 event of five colors, which is similar to the two-color Ellsberg paradox. For unlikely events (one and three colors), a-insensitivity has an effect contrary to ambiguity aversion, resulting in ambiguity seeking. We indeed found that over 80% of the subjects exhibited ambiguity seeking there. This finding confirms Ellsberg's prediction made in the 1960s (see Ellsberg 2001, pp. 203, 205–206) and agrees with common empirical findings (Trautmann and van de Kuilen 2015). Chew et al. (2017) found it (preference for skewed ambiguity) in a study on source preference for different kinds of probability intervals and their unions. Combined with ambiguity aversion for likely events, it gives an estimated a-insensitivity index of 0.81, showing that this component is also present in the traditional Ellsberg setting. Our study adds to several other recent studies questioning the universality of ambiguity aversion (Tversky and Kahneman 1992).

We incorporated the second (week) and third (year) treatment to see whether changes to outcomes, keeping the source fixed, affected ambiguity attitudes. Our analyses confirm that they do not, as shown by their matching probability curves (see Figure 5), the comparisons of ambiguity attitude per event (see Table 2), the strong correlations between these treatments (see Table 4), similar fits of parametric functions (see Table 6), and same best-fitting parameters (see Table 7). This is further confirmed by the same violations of monotonicity and correlations between indexes (see Table 5). Because these anticipated results are based on accepted null hypotheses, we used two treatments so as to have high statistical power.

We incorporated the fourth (kid) treatment to acquire more natural, although not yet entirely natural, uncertainty. We made a special effort to increase source preference by making this treatment what can be called a feel-good treatment. The general charitable context may have generated extra general utility. However, this utility is the same for all gambles and does not affect, nor is affected by, decisions. In particular, it does not have any impact on the kid. Decisions can be affected, and this can be modeled, as follows, though. Being affected by the uncertainty about the child, as opposed to certainty, is liked more, even if it does not affect the (monetary) outcomes or their likelihood. This is best modeled through an elevated weighting function and source preference, in the same way as this happened with basketball uncertainty for basketball fans in Tversky and Fox (1995). We added the fifth (health) treatment as the most deviating one, with the most natural source of uncertainty and again different outcomes. It could be called an understand-good treatment. The fourth and fifth treatments exhibit increased source preference and sensitivity through their matching-probability curves (see Figure 5) and eventwise ambiguity attitudes (see Table 2). Their aversion and insensitivity indexes are less related to the other three treatments, with (a) the cognitive sensitivity index of the health treatment even being unrelated to those of the first three treatments (see Table 4),[13] (b) their monotonicities being increasingly better than for the first three treatments, (c) their overlap being lower because there was less irrationality to be shared (see Table 5), (d) better fits of parametric families suggesting less noise (see Table 6), and (e) parameters from the fitted families (see Table 7) deviating from the first three treatments.

Outcome-based ambiguity models, such as the smooth model (Klibanoff et al. 2005), cannot accommodate one of the main empirical findings, the fourfold pattern. For gains, this pattern entails ambiguity aversion for moderate and high likelihoods but ambiguity seeking for low likelihoods. That is, we have within-source event dependence of ambiguity aversion. This cannot be accommodated through outcomes. Because we found this within-source event dependence in all our measurements, and because it is also the common finding in the literature (Trautmann and van de Kuilen 2015), we did not try to fit our data using outcome-based ambiguity models, but we used the event-based ambiguity models comprised in the biseparable model. It is then still empirically possible, a priori, to find that ambiguity attitudes depend on outcomes, and vary for different outcomes. We thus tested for this outcome dependence, besides (between-)source event dependence, and found the latter but not the former.

Although many more studies are needed before general conclusions can be drawn, our results suggest that ambiguity attitudes depend on the sources of uncertainty (the kinds of events) more than on outcomes. This finding supports empirical theories that model ambiguity attitudes through event functions. Such theories include Choquet expected utility, multiple priors and α-maxmin models, new prospect theory, and their many recent generalizations, as well as biseparable utility as used in our analyses. Our findings are consistent with Abdellaoui et al. (2016) and (2017). They did not investigate dependence of ambiguity attitudes on outcomes as we did but instead investigated dependence of utility of money on the source of uncertainty. They did not find such dependence.

We have demonstrated how the full richness of ambiguity can be investigated. Of course, completing this large task is impossible for one paper. Even a complete design of all combinations of the sources and outcomes that we considered in this study would be too large for one paper. We chose the combinations that we expected to give the most interesting results at this stage.

One reason for us to consider waiting time as outcome is that there is much interest in the effect of ambiguity on optimal stopping times. See Della et al. (2014), Miao and Wang (2011), Nishimura and Ozaki (2007), and Riedel (2009). The results of these studies could have been distorted if ambiguity attitudes toward waiting-time outcomes were different from other outcomes. It is therefore reassuring that we find no such difference.

Our deviating findings for the kid treatment are unsurprising given that we deliberately induced positive emotions for the events involved, which is comparable to the emotion-arousing outcomes of Rottenstreich and Hsee (2001). In this sense, our finding is similar to Tversky and Fox (1995), whose finding of ambiguity seeking for ambiguous basketball events under basketball fans is similarly unsurprising.

The high sensitivity in the health treatment and the absence of ambiguity aversion are remarkable. Subjects discriminated between different levels of likelihood considerably better than in the other treatments. It suggests greater interest and better motivation on the part of subjects, even though this treatment could not satisfy the real incentive principle of experimental economics. It has been observed before that subjects are well motivated to answer questions about health, even if hypothetical (see the end of Section 2 in Bleichrodt and Pinto 2009). In the same spirit, many people voluntarily donate money to support medical investigations. One reason for the reduced ambiguity aversion could be that outcomes in this treatment were perceived as losses. It is well known that there is less ambiguity aversion for losses (Attema et al. 2013, Trautmann and van de Kuilen 2015).

Closest to our data fitting of ambiguity are Ahn et al. (2014) and Hey et al. (2010). These studies compared different general ambiguity theories regarding their overall fitting and predictive power. Our study focuses on biseparable utility but is more general than the preceding studies in comparing different parametric models, distinguishing between several components of ambiguity attitudes, and considering a rich domain.

7. Conclusion

Following the recommendation of Ellsberg (2011) and many other authors, we investigated ambiguity and its richness empirically, with varying outcomes, varying uncertain events, and combinations of both. The richness considered and the use of natural events reinforces the external validity of our general findings. These findings are as follows:

- For natural uncertainties, ambiguity aversion is less pronounced, and rationality (sensitivity and monotonicity) is higher than for artificial Ellsberg uncertainties.
- Ambiguity attitudes are more driven by the kind of uncertainty than by the kind of outcome.
- Our two indexes of ambiguity attitudes capture most of the variance in the data.
- Insensitivity (inverse S)[14] is even more important for ambiguity than for risk, implying that ambiguity seeking is prevailing for some stimuli.
- Individual ambiguity attitudes have predictive power across different sources.
- The Goldstein and Einhorn (1987) family of weighting functions serves well to model ambiguity attitudes.

Several specific findings in this paper depend on the particular sources and outcomes considered. Future studies will further investigate the relevant phenomena of ambiguity attitudes in different contexts, bringing new insights into this important and new domain of human decision making.

Appendix. Experimental Instructions (Screenshot)

Figure A.1. Screenshot of Health Treatment

Imagine that you are diagnosed with a certain disease. You have to receive a treatment against the disease; there is no possibility to abstain from treatment. The only choice you have is which treatment you will receive.
Research on the disease all over the world has revealed the following facts: There are ten possible viruses causing the disease (i.e. virus 1, virus 2, virus 3, virus 4, virus 5, virus 6, virus 7, virus 8, virus 9, and virus 10). The prevalence (the rate of occurrence) of the viruses causing the disease is unknown. And there is no way to diagnose which virus you have; they all lead to the same disease. (The viruses are mutually exclusive; you will always have just one virus). Only if the real virus is treated will the disease be totally cured. Assume that then you will live **50 years** longer from now on in good health and die. If the real virus is not treated, you will live only **1 year** longer from now on in good health and die.

For all the decision scenarios in this part, there are two possible treatments. Both treatments have the **same treatment duration** and the **same costs**. Neither treatment has **adverse side effects**.
Treatment K:
Treatment K treats the disease with a **K**nown success rate. The success rate is known from experiences with previous patients. It uses a broad-spectrum antiviral supplement, which is not specific to any one of the viruses, but is generally effective for all viruses alike. For example, for treatment K, the success rate can be 10%. (As an explanation: if 10 out of 100 patients are cured, the success rate is 10%.) We will also consider other possible success rates. If you are cured by treatment K, you will live 50 years longer from now on in good health; otherwise you will live only 1 year longer from now on in good health.
Treatment U:
Treatment U is new. It uses ten different supplements. We name the ten supplements S1, S2, S3, S4, S5, S6, S7, S8, S9, and S10 respectively. Each supplement is effective for the corresponding virus. (For example, supplement S7 is only effective for virus 7.) However, different supplements are not always available. You will therefore be treated only with the available supplements. Remember that there is no way to tell which virus causes your disease. If the right supplement for the real virus is chosen, then you will be cured and live 50 years longer from now on in good health; otherwise you will live only 1 year longer from now on in good health.

Endnotes

[1] See Abdellaoui et al. (2011, 2017), Baillon and Bleichrodt (2015), Baillon et al. (2017), Chew et al. (2008, 2012), Fox and Weber (2002), Fox et al. (1996), Fox and Tversky (1998), Li (2017), and Tversky and Fox (1995).

[2] Because their outcomes differed regarding correlations, the underlying uncertainty also differed. Ambiguity neutrality or ambiguity aversion could therefore not be calibrated.

[3] We use the term "urn" in this paper because it is customary in the field. For subjects during the experiment (and in screenshots as in Figure 3), we used the term "bag."

[4] For swiftly implementing the composition of urn K, for every color, groups of 20 balls were stringed (the balls had holes), thus enabling us to quickly and reliably prepare any number of balls between 0 and 100 in front of the participants, verifiable for everyone.

[5] For $j = 5$, we thus obtained two measurements of $m(5/10)$. These were never statistically different for any treatment (Wilcoxon signed rank tests), suggesting that there were no framing effects or color preferences. In most of the following analyses, we therefore used the average of the two observations of $m(5/10)$. In parametric fittings, it is appropriate to take the two observations as separate, and so we did.

[6] Unfortunately, permission could not be obtained from the Bharti foundation to reproduce the photo.

[7] Note that the outcome is money (€500 or €0) to be paid to the subject of the experiment just as in the basic treatment. There was no payment to the charity. We only use the context to create uncertainty as described in the following paragraph.

[8] This source preference is similar to the source preference of basketball fans for basketball uncertainty (Tversky and Fox 1995).

[9] All sessions were scheduled on the same day to avoid the scenario that participants could know that a charitable program in rural India was involved and could have gathered information about it.

[10] By our middle-point approximation of matching probabilities, ambiguity neutrality with $m(j/10) = j/10$ can never happen exactly (except for the average of the two measurements for a-neutral probability 0.5). Hence, we also considered the modification of Equation (3) into $|j/10 - m(j/10)| \leq 0.005$ and did statistical tests using two-sided Wilcoxon signed rank tests and comparing individual AAj's with 0.005 and −0.005, taking as p-values the larger of the two tests. This modification did not seriously affect our results. Only the significance levels of (0.5, year), (0.1, health), and (0.9, health) were downgraded by one asterisk.

[11] We also extracted the two indexes b and a for every subject per treatment using linear least squares estimations. See Online Appendix OA.4 for medians and comparison among treatments. They confirm all results reported here.

[12] We also fit these parametric families individually. For medians of those individual parameters, correlations of parameters across treatments, and correlations among parametric families per treatment, see Online Appendix OA.7. They confirm all results reported here.

[13] The correlation for the aversion parameter b of the year and kid treatment is high—though still between the Ellsberg-urn-type ones and the others—probably because they both involve long-term non-consequential feel-good money/education. Other than this, all the highest correlations are for similar events, confirming that events affect ambiguity attitudes more than outcomes do.

[14] Or, equivalently (but without an analog for risk), perceived level of ambiguity in the α-maxmin model.

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