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A Criticism of Doyle's survey of time preference: A correction regarding the CRDI and CADI families

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Abstract

Doyle's (2013) theoretical survey of discount functions criticizes two parametric families abbreviated as CRDI and CADI families. We show that Doyle's criticisms are based on a mathematical mistake and are incorrect.

Keywords: intertemporal choice, constant relative decreasing impatience, hyperbolic discounting.

1 Background and analysis

Doyle (2013) provides a useful theoretical survey of the most popular discount functions for intertemporal choice. Unfortunately, his analyses of the CRDI (constant relative decreasing impatience) and CADI (constant absolute decreasing impatience) families of Bleichrodt, Rohde, and Wakker (2009; BRW henceforth) are incorrect. Let D denote the discount function and let T denote time. The CRDI family is defined as follows, with $\beta > 0, \rho > 0$, and $\psi \in \mathbb{R}$ denoting parameters:

$$\text{If } \Psi > 0, \text{ then } D(T) = \beta.exp(-\rho T^{\Psi}) \text{ for } T \geq 0; \quad (1)$$

$$\text{If } \Psi = 0, \text{ then } D(T) = \beta.T^{-\rho} \text{ for } T > 0; \text{ D is not defined for } T = 0; \quad (2)$$

$$\text{If } \Psi < 0, \text{ then } D(T) = \beta.exp(\rho T^{\Psi}) \text{ for } T > 0; \; D \text{ is not defined for } T = 0. \quad (3)$$

The CADI family is defined by replacing T by $exp(T)$ in the right-hand sides of the equalities in Eqs. 1-3, but we focus on CRDI here. Doyle wrongly assumes the normalization

$$D(0) = 1 \quad (4)$$

for all families that he considers. This normalization has often been assumed in the literature, but it should not be used in Eqs. 1-3, as pointed out by BRW (p. 29 l. 2 ff). For Eq. 1, $D(0) = \beta$ immediately follows from substitution, invalidating Eq. 4. For Eqs. 2 and 3 we have $\lim_{T\downarrow 0} D(T) = \infty$ (BRW 2009 p. 32), again invalidating the extension in Eq. 4. The normalization in Eq. 4 can be obtained in Eq. 1 by setting $\beta = 1$. But then, obviously, $\beta = 1$ should be consistently set for all T, including all $T > 0$. Such a consistent normalization does not affect preference. The normalization of Eq. 4 cannot be obtained for Eqs. 2 and 3.

Doyle assumes Eq. 4 for Eq. 1, but inconsistently does not set $\beta = 1$ for $T > 0$. He also erroneously assumes Eq. 4 for Eqs. 2 and 3. He does not state his assumed Eq. 4 explicitly, but all his results and conclusions about the CRDI and CADI families essentially use it (see our Appendix) and, hence, are invalid. For example, contrary to Doyle (p. 127 following Eq. 31), the β parameter in CRDI is not the β parameter in the β-δ model, but is just a normalization parameter with no empirical meaning.[1] Further, contrary to Doyle's implicit assumption throughout, present values do not exist for Eqs. 2 and 3.

The CRDI family D is simply the transform $D = exp(-U)$ of the CRRA utility family U for expected utility, which is one natural family (Wakker 2008) rather than "three distinct sub-models" (Doyle's end of Section 3.6.2). Doyle's mistake in setting $D(0) = 1$ in Eqs. 2 and 3, for instance, is the same as wrongly assuming $U(0) = 0$ for the negative-power $U(x) = -x^{\Psi}$ for $\psi < 0$ or for the logarithmic $U(x) = \ln(x)$. It should be $U(0) = -\infty$, because otherwise monotonicity (and continuity) are violated.

BRW explain that their CRDI family has the flexibility to capture all possible degrees of increasing and decreasing impatience, unlike any other currently popular discount family. Hence, their CRDI family can accommodate the full range of individual time preferences, including subjects whose deviations from constant discounting are extreme. It can therefore serve well to fit data at the

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[1] See BRW (p. 31 last line and Definition 4.3), who use k to denote Doyle's β. Doyle (p. 127 bottom), strangely enough, cites this text of BRW, but still maintains his incorrect interpretation of β.

individual level. We hope that the family will be used despite the incorrect criticisms by Doyle.

References

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Appendix: Reproducing Doyle's results

Doyle does not explicitly state his use of the false Eq. 4, but it can be inferred from his incorrect Eqs. 31-36. It also explains his incorrect interpretation of the β parameter. Because multiplying all discounted utilities by the same factor $\beta > 0$ does not affect preference, the readers can immediately see that β is just a normalization factor with no empirical meaning in Eqs. 1-3.

We provide an algebraic derivation of our claim about Doyle's implicit assumption for the most complex case, Doyle's Eq. 33. We explained in the main text that neither Doyle's Eq. 33 nor any close analog can be derived, because present values do not exist. We next show that with the incorrect Eq. 4 and thus, in particular, with the incorrect assumption that present values exist, Doyle's Eq. 33 readily follows. This reveals that Doyle's analysis indeed incorrectly assumes Eq. 4. For the other cases the analysis is similar but simpler. Doyle's Eq. 33 concerns our Eq. 3 ($\psi < 0$), and is reproduced next:

$$\rho = \frac{\ln(\beta F/P)}{-T^{\Psi}}. \tag{5}$$

In all the analyses relevant here, Doyle assumes that utility is the identity ($U(x) = x$). He assumes, with $(T : F)$ denoting receiving $\$F > 0$ at time $T > 0$, and P denoting present value (wrongly assumed to exist, as an implication of Eq. 4):

$$(0 : P) \sim (T : F). \tag{6}$$

If we apply Eq. 4, $D(0) = 1$, to the left-hand side but Eq. 3 to the right-hand side, then we get:

$$P = F.\beta.exp(\rho T^{\Psi}). \tag{7}$$

Eq. 5 now follows:

LEMMA 1

For Eq. 3 with Eq. 4 ($D(0) = 1$), Eq. 6 can be satisfied, and it implies Eq. 5.

Proof: Consider the following rewritings of Eq. 7:

$$\frac{\beta F}{P} = \frac{1}{exp(\rho T^{\Psi})} = exp(-\rho T^{\Psi});$$

$$\ln(\frac{\beta F}{P}) = -\rho T^{\Psi};$$

$$\frac{\ln(\beta F/P)}{-T^{\Psi}} = \rho. \qquad \square$$

We briefly comment on Eq. 1 and Doyle's related Eq. 31. Doyle's equation is correct if $\beta = 1$ is assumed, i.e., if β is dropped. This can be proved in two ways:

(1) If one normalizes Eq. 1 by setting $\beta = 1$ and uses the normalized $D(0)$ in Eq. 1;

(2) If one does not normalize, taking the nonnormalized Eq. 1 with general $\beta > 0$ in Eq. 1 and replacing Eq. 4 by the nonnormalized $D(0) = \beta$. In both proofs, β drops out, which is unsurprising because it should have no empirical meaning. However, one cannot use the normalized D in Eq. 4 together with the nonnormalized Eq. 1 with a general $\beta > 0$. The latter is what Doyle does to derive his Eq. 31, using algebra similar to the above proof.