Corrected Proof of Lemma 3 (p. 496) of
Itzhak Gilboa, David Schmeidler, & Peter P. Wakker (2002), "Utility in Case-Based Decision Theory," *Journal of Economic Theory* 105, 483–502.

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As pointed out to me by Han Bleichrodt, the proof, incorrectly, assumes that the projection of a closed set is closed. This need not hold true in general. For example, within $\mathbb{R}^2$, project the graph of the function 1/x for positive x (i.e. the set $\{(x,1/x): x > 0\}$) on the x-axis. The graph is closed but its projection is the open, and not closed, set $\{x\in\mathbb{R}: x>0\}$ of positive numbers. The lemma is correct though, and V is continuous. A different proof is given hereafter.

Consider the set $V^{-1}\{\beta\in R|\beta \succ \alpha\}$, and let $(x_2,\ldots,x_n)$ be an element thereof. Then $(\gamma,x_2,\ldots,x_n) \in I_{ab}$ for some $\gamma \succ \alpha$, and, by monotonicity and favorableness of problem 1, a $\prec_{(\alpha,x_2,\ldots,x_n)}$ b. For illustration, assume that problem 2 is favorable. By preference continuity and connectedness, either x_2 is maximal (a case that is actually excluded by the other axioms, especially solvability, but we will not prove this) or there is an $x_2' \succ x_2$ such that still a $\prec_{(\alpha,x_2',x_3,\ldots,x_n)}$ b. For illustration, assume further that problem 3 is unfavorable. By preference continuity and connectedness, either x_3 is minimal (which is actually excluded by the other axioms) or there is an $x_3' \prec x_3$ such that still a $\prec_{(\alpha,x_2',x_3',x_4,\ldots,x_n)}$ b. We end up with an inductively defined neighborhood of $(x_2,\ldots,x_n)$ in $V^{-1}\{\beta\in R|\beta \succ \alpha\}$ of the form $B_2 \times \ldots \times B_n$ where for each j:

$B_j = \{\delta: \delta \prec x_j'\}$ for an $x_j' \succ x_j$ if problem j is favorable and x_j is not maximal.

$B_j = \{\delta: \delta \succ x_j'\}$ for an $x_j' \prec x_j$ if problem j is unfavorable and x_j is not minimal.

$B_j = R$ if problem j is neutral, or if problem j is favorable and x_j is maximal, or if problem j is unfavorable and x_j is minimal.

For every element of $V^{-1}\{\beta\in R|\beta\succ \alpha\}$ we can construct a neighborhood within $V^{-1}\{\beta\in R|\beta\succ \alpha\}$, so that the latter set must be open. Similarly, $V^{-1}\{\beta\in R|\beta\prec \alpha\}$ is open for each α. Continuity of V follows. □