Suggestions for Presenting Tradeoff-Consistency Preference Conditions

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This note eplains how tradeoff consistency conditions can be explained didactically and transparently. We consider preference conditions of the kind

$(a_1,x_2) \sim (b_1,y_2)$, $\quad$ $(a_1,x_2) \sim (b_1,y_2)$, &
$(c_1,x_2) \sim (d_1,y_2)$ $\quad$ imply
$\quad$ $(a_1,v_2) \sim (b_1,w_2)$.

First, such conditions with indifferences are way more easy to understand, and convey, than with preferences, mainly because one does not have to remember directions of preferences. Köbberling & Wakker (2003) first used such a version with indifference. Well, more precisely, Wakker (1988) used it in a model without monotonicity so that weights could be negative.

I learned that presenting it as above, with four indifferences involved, is complex for people to immediately grasp. One can almost halve the cognitive burden by first presenting only two indifferences:

$(a_1,x_2) \sim (b_1,y_2)$ &
$(c_1,x_2) \sim (d_1,y_2)$

using those to convey the intuition of strength of preference, that its interpretation is that the strength of preference of a_1 over b_1 is as strong as that of c_1 over d_1. Or improvement of a_1 into b_1, or tradeoff. The outcomes x_2, y_2 may be called gauge outcomes. Let the audience grasp this intuiton from only two indifferences, and they get it. The above layout, with corresponding symbols right below each other, helps to see the idea.

In case of heavy use, can introduce notation such as $[a_a;b_1]$ or $a_1 \ominus b_1$, and write

$a_1 \ominus b_1 \sim^t_1 c_1 \ominus d_1$

if it is for a coordinate-dependent representation $V_1 + \ldots + V_n$, or write

$a_1 \ominus b_1 \sim^t c_1 \ominus d_1$

if it is for a weighted utility representation $p_1U + \ldots\ p_nU$.

If people grasp the two-indifference idea, the war is won. The above preference condition can then be stated easily and completely verbally:

"improving any outcome in a $\sim^t$ relationship breaks that relationship."

It can also be $\sim^t_1$, depending on context.

People who are aware of this basic technique and used it on several occasions include Karni, Prelec, and Tversky. (Prelec was a student of Luce and got exposed to the techniques of KLST 71 at young age-Karni may have taken it from me.) In my early years, I used inefficient formulations such as

"the strength of preference of receiving a_1 instead of b_1 has been revealed to be as strong as the strength of preference of receiving c_1 instead of d_1."

Prelec in similar situations needed fewer words:

"a_1 is to b_1 what c_1 is to d_1."

That for most preference conditions, versions with indifferences suffice, can be derived from Wakker (1989), Theorem III.6.6 (p. 70), Statement (ii), together with Remark III.7.3. The only nonindifference condition needed is weak separability, which for monetary outcomes is implied by monotonicity. Other than that, for two nonnull coordinates one needs the hexagon condition which only involves indifferences. For more than two nonnull coordinates Statement (ii) puts up CI (coordinate independence, which is the sure-thing principle, or preference separability), a condition that involves preference and not just indifference. Remark III.7.3 however shows that, given weak separability, only the version of that condition with indifferences is needed. This way conditions with only indifferences give additive representability. Usually, whatever more is needed in particular theorems is not very difficult to do.

References

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