

1 This paper discusses the role of preference axioms in expected utility, building on previous
2 discussions in this journal. The independence axiom, if accepted, provides the proper terms
3 (probabilities and utilities) for risky decisions, without specifying what values those terms
4 should take. Thus, while necessary for rationality, it can never be sufficient. Expected utility
5 concerns the aggregation over different resolutions of uncertainty. It does not concern
6 aggregation over different persons (welfare and policy making), different time points (repeated
7 decisions), or different attributes (e.g., duration versus quality of life). People can deviate from
8 expected utility for normative ("rational"), psychological ("irrational"), and tractability reasons.
9 We believe that discrepancies between expected utility and behavioral decision making, and
10 the resolution of those, are an essential part of expected utility's prescriptive contribution.

11

12 *Key words:* Expected utility, independence, preference axioms

1 This paper studies the role of preference axioms in expected utility. Implications for issues
2 about the normative status of expected utility, raised before in this journal, are described. Such
3 issues concern single-shot versus repeated decisions, single-person versus group decisions,
4 normative versus descriptive, internal consistency versus goal-fulfilment, axiomatic
5 justifications versus axiomatic criticisms, and the persistence of descriptive deviations.

6 Decision analysis is based on the expected utility approach to decision making under risk.
7 Its use in medicine has been criticized on several grounds, such as lack of relevance for daily
8 practice, spurious quantifications of lives versus money, and dehumanization of patient care.
9 In short, it "takes the heart out of clinical judgment".¹⁻³ Furthermore, decision analysis has not
10 been widely accepted among clinicians, and policies based on decision analyses have sometimes
11 been unconvincing to clinicians.^{4,5}

12 Cohen⁶ adds another objection to decision analysis by criticizing expected utility as a
13 normative theory. His main criticism is that the rationality of the theory cannot be founded on
14 axioms such as independence. The theory needs external justification in terms of goals to be
15 fulfilled. Cohen is not alone in criticizing expected utility theory for this reason.^{7,8} In
16 subsequent comments to Cohen's discussion, several other insights in the normative status of
17 expected utility have been put forward.⁹⁻¹³ Our paper aims to provide further arguments,
18 building on the mentioned references, and in reply to the editor's comment.¹⁴

19 Although most of this paper aims to clarify the arguments for and against expected utility
20 without choosing side, let us state our subjective viewpoint. We believe that expected utility
21 should be the normative basis for medical decision making.

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23

24 The Preference Axiomatization of Expected Utility

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26 In most of the discussion we restrict attention to the case of known objective
27 probabilities. Von Neumann and Morgenstern¹⁵, and others after them, provided a preference
28 axiomatization for expected utility. The main axiom is "independence." (For its history, see

1 Fishburn & Wakker.¹⁶⁾ Independence says, loosely speaking, that, in a situation where with
 2 probability p you receive some gamble and with probability $1-p$ "something else," you prefer
 3 replacement of the gamble by any higher-preferred gamble, irrespective of what "something
 4 else" stands for. The remaining preference axioms in the following theorem are not central for
 5 the ensuing discussion, hence no definition is provided here. Definitions can be found in the
 6 literature.^{7,15,17}

7

8 PREFERENCE AXIOMATIZATION OF EXPECTED UTILITY. The following two statements are
 9 *equivalent* for preferences over gambles:

10 (i) There exists a utility number for every outcome so that gambles are valued by their expected
 11 utility.

12 (ii) The preference relation satisfies the following four conditions: (a) independence, (b)
 13 transitivity, (c) complete comparability, (d) continuity.

14

15 *Equivalence* of statements means that one statement holds true if and only if the other
 16 does, that is, the statements say the same thing in different words. The axiomatization says
 17 that, whenever the preference relation of a person satisfies independence and the other
 18 preference conditions, then the person behaves in accordance with expected utility. Whenever
 19 the person violates one of the preference conditions, then expected utility theory does not hold
 20 for that person. Conversely, whenever the preference relation of the person can be described
 21 by expected utility, the four preference conditions are satisfied and, whenever the preference
 22 relation of the person cannot be described by expected utility, then at least one of the
 23 preference conditions must be violated.

24 An axiomatization simplifies the justification of a model. Justifying expected utility as in
 25 Statement (i), without a preference axiomatization as in Statement (ii) available, is difficult. It
 26 is not clear where the utility numbers come from or what they mean, and it is not clear why
 27 these numbers should be used in the expected utility formula as they are, and not in different
 28 formulas. The described problem for Statement (i) is more basic than its quantitative model

1 being appealing or not. The problem lies in the very meaning. The empirical content of the
 2 statement, without Statement (ii) available, is not clear and therefore is not easy to judge,
 3 either for or against. Without Statement (ii) available, the claimed model seems ad hoc.

4 Statement (ii) solves the mentioned problem of Statement (i). It does have direct
 5 empirical meaning because its conditions are directly stated in terms of choice, the observable
 6 empirical primitive of decision making. One can imagine what the conditions mean, and
 7 therefore one can be convinced to accept or reject the conditions.

8 In summary, a preference axiomatization is a logical result, showing logical equivalence
 9 of two statements. The first statement describes the existence of a quantitative theoretical
 10 model, the second "translates" the first into the empirically meaningful language of preference
 11 and choice.

12 We have stated at length what the theorem does in order to prepare for a discussion of
 13 what the theorem does *not* do. Several misunderstandings have arisen in the literature because
 14 meanings were assigned to the axiomatization that are not there. Hence a discussion of what
 15 the axiomatization does *not* do seems warranted. To that we now turn.

16

17

18 The Preference Axiomatization of Expected Utility: What It 19 Does Not Do

20

21 GOAL-ORIENTEDNESS AND EXTERNAL CRITERIA: UTILITIES REMAIN TO BE
 22 CHOSEN

23

24 Assume for now that we have been convinced of the preference conditions of Statement
 25 (ii), thus of expected utility as in Statement (i). As strong as Statement (i) is in claiming the
 26 existence of utility numbers, so weak it is in specifying what those numbers should be, and how
 27 they should be determined (Broome¹⁸, Section 5.1). Obviously, utilities are fully determined by
 28 answers to standard gamble questions. However, what those answers should be, and therefore

1 what the utilities should be, remains unspecified. In other words, the preference
2 characterization provides the expected utility language for decision under risk, without
3 prescribing the content of that language. It shows that probabilities and utilities are the
4 relevant terms without showing what the specific content of those terms should be. It
5 describes the "syntax" of the language, i.e., its general rules, but it does not describe the
6 "semantics," i.e., its content. In medical decision making, the content should obviously
7 describe medical factors and knowledge.

8 Cohen⁶ alludes to the relevance of semantics by pointing out that expected utility
9 applications need a justification in terms of external criteria. Such criteria are called goals of
10 the decision maker by Baron. In medical decision making, the goal is optimization of the
11 health of patients. If expected utility is only part of rational goal-fulfillment and not all of it,
12 then, contrary to what is sometimes suggested⁶, it cannot yet be concluded that violations of
13 expected utility are rational. External criteria of goal fulfillment come in addition to, and not
14 instead of, the axioms (Broome¹⁸, Section 1.4). As pointed out by Wu¹⁰, advocates of
15 expected utility consider independence as necessary for rationality, but not sufficient. In the
16 medical area, rationality cannot be achieved without medical knowledge.

17 18 JUSTIFICATION OR CRITICISM OF EXPECTED UTILITY

19
20 An axiomatization by itself does not entail a justification. By clarifying the empirical
21 meaning of a model, it as well aids critics of the model as advocates. Indeed, the independence
22 condition of von Neumann and Morgenstern has not only been used to defend expected utility
23 but as well to criticize it.^{19,20}

24 Not only can the preference conditions in Statement (ii) be used to justify or criticize
25 expected utility in Statement (i), but it can also be the other way around. For example, Cohen⁶
26 first criticizes expected utility and then uses that criticism against independence. It is
27 nevertheless more natural to use preference conditions as in (ii) to justify or criticize a model
28 than the other way around because preference conditions have a clearer empirical meaning.

1 Let us briefly mention that axiomatizations of expected utility through other conditions
2 than independence exist in the literature. Nease¹² already mentioned the logical equivalence of
3 independence and another similar "substitution" axiom. That substitution axiom was also used
4 in other works.²¹⁻²³

5 Initiated by Hammond²⁴, appealing justifications underlying independence have been
6 developed during the last years that are based on dynamic decision principles. Such dynamic
7 principles underlie any decision tree analysis and are followed almost unconsciously by all of
8 us. Hammond made the surprising discovery that such principles, while almost self-evident,
9 have much logical force. They turn out to *imply* that the independence axiom and that
10 expected utility must hold.²⁴ Hence, those dynamic principles can provide an alternative
11 justification for expected utility that underlies independence. In spite of the high interest of
12 Hammond's discovery, a discussion of his ideas here lies outside the scope of this paper. It is
13 interesting, however, that much of Hammond's dynamic arguments can be recognized in
14 Baron's⁹ second part. Dynamic choice principles (preferences should be consistent over time if
15 the situation does not change significantly) seem to be central in his argument, maybe even
16 more than the goal-oriented interpretation of preference.

18 SINGLE-SHOT VERSUS REPEATED DECISIONS

20 One point of confusion in the literature concerns the question of whether expected utility
21 can only have meaning in repeated decisions or whether it should be applied to single-shot
22 decisions. The issue was raised in this journal by Cohen⁶, who considered expected utility a
23 "long-run property of a random variable." A similar argument was put forward by Lopes.²⁵
24 Expected utility is, however, basically defined for single-shot decisions.^{10,11,26,27} The
25 misunderstanding may have arisen by an erroneous extension of the frequentist interpretation
26 of probability to a "frequentist" interpretation of decision. Under the frequentist interpretation
27 of probability, probabilities can only be assigned to repeatable events and are defined as
28 limiting frequencies. Then, obviously, the decisions in which such events partake need not be

1 repeatable. Even if it is possible to repeat tosses of a coin as often as desired, a gamble on a
2 toss may be a one-time opportunity.

3 Whereas expected utility is basically defined for single-shot decisions, applications to
4 repeated decisions are not excluded and are even warranted. They require, however, further
5 assumptions on repetitions and evaluations of sequences of outcomes, in addition to the pure
6 expected utility assumption (Broome¹⁸, Section 11.1).

7 In fact, if a decision is repeated often and sequences of outcomes are to be evaluated by
8 their average or total, then expected utility, as well as any of its alternatives, will recommend
9 expected value maximization for each individual choice. That is a simple consequence of the
10 law of large numbers. In such situations expected utility in the individual choice is not
11 relevant, quite contrary to the suggestion sometimes made in the literature that expected utility
12 be only meaningful in such situations. The phenomenon was mentioned by Samuelson²⁸ in
13 relation to a colleague who refused a single fifty-fifty gamble with outcomes \$200 and -\$100,
14 but said he would accept the gamble when repeated several times. Also Edwards²⁹ called
15 attention to this phenomenon. A useful discussion of the phenomenon, in particular of the
16 assumption that average or total should be maximized, is presented in Section 4.1 in Broome.¹⁸

17

18 SINGLE-PERSON VERSUS GROUP DECISIONS

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20 The implications of expected utility for group decisions have also been debated.
21 Whereas expected utility is basically defined for single preference relations, applications to
22 groups of people are not excluded and are even warranted, for instance in policy decisions.
23 They require, however, further assumptions about group aggregation, in addition to the
24 expected utility assumption.

25 A famous result is Harsanyi³⁰. Once his modeling assumptions are accepted, a simple
26 separability condition (individual orderings of gambles are independent of gambles for other
27 individuals) implies utilitarianism. An appealing similar result is provided in Section 10.2 of
28 Broome.¹⁸ Alternatively, an egalitarian Rawlsian approach can be used, or the libertarian

1 approach or freemarket approach.³¹ These are three different ways for aggregating individual
2 preferences into group preferences and can all be reconciled with the expected utility dictum.
3 Contrary to what has sometimes been suggested in the literature, expected utility does not
4 determine group preferences and in particular it does not impose utilitarianism.

5

6 NORMATIVE VERSUS DESCRIPTIVE APPLICATIONS

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8 Another issue of debate is whether the expected utility axioms are relevant to normative
9 or descriptive applications. The answer is that the characterization by itself does not commit
10 to normative or descriptive applications. Normatively, one can be convinced of the
11 appropriateness of expected utility if and only if one can be of the axioms, and one can criticize
12 expected utility if and only if one can criticize the axioms. Descriptively, one can demonstrate
13 that expected utility is violated to a large degree if one can demonstrate that the axioms are
14 violated to a large degree, and one can argue that expected utility is satisfied to a sufficient
15 degree if one can argue that the axioms are satisfied to a sufficient degree.

16

17 Violations of Expected Utility

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19 We briefly discuss the implications of the above analysis for violations of expected utility
20 other than for the normative reasons already mentioned. Descriptively, most people are not
21 aware of the expected utility model and the independence condition, and many psychological
22 concerns of a different nature come into play. Examples are framing, regret, anxiety,
23 disappointment, etc.³²⁻³⁶

24

25 Another cause of deviation is that people do not have pre-defined and well-articulated
26 values and utilities. Indeed, if the preference conditions of the characterization theorem have
27 been accepted and thus the syntax of expected utility, then the semantic questions still remain
28 open. Difficulties at the semantic level then can still generate deviations. These points are at
the heart of the "constructive" interpretation of preference which holds that preference

1 elicitation is a constructive process rather than a measurement process.^{37,38} That viewpoint
2 implies that inconsistencies in utility elicitation should not just be accepted as they are, but
3 more interactive work should be done to correct them and to construct sensible preferences
4 that truly represent the patients' interests.

5 Arguments of tractability and calculation costs are ignored in preference axiomatizations.
6 They therefore provide another source of deviation from expected utility.^{39,40} Tractability
7 arguments can also support expected utility. That point was put forward by Eeckhoudt¹¹, who
8 thinks that expected utility is not normative but recommends its use as long as a more tractable
9 alternative is not available. Indeed, it should be understood that no currently-existing decision
10 tree program can be used if expected utility is abandoned.²⁴ Also Nease¹² pointed out that
11 even if the expected utility axioms are not fully normative, still the expected utility technique
12 can provide useful insights.

15 Conclusion

16
17 Preference axioms "translate" the theoretical meaning of a quantitative model into conditions
18 directly in terms of preferences, that is, conditions with direct empirical meaning. The axioms
19 simplify both justifications of a model and criticisms. Thus, von Neumann-Morgenstern
20 independence has been used both to justify expected utility and to criticize it. If accepted as
21 rational, preference axioms show what the appropriate terms and parameters are for making
22 rational decisions (probabilities and utilities in the case of expected utility). Preference axioms
23 therefore provide the syntax of a rational language. They do not provide the semantics.
24 Semantics should involve the goals of the decision maker, and for instance invoke medical
25 knowledge in medical decision making.

26 The basic expected utility model and its axioms consider single-decision maker single-
27 shot decisions. They speak to the aggregation over different resolutions of uncertainty and not
28 to the aggregation over different persons, or the aggregation over repeated choices and

1 different time points, or, similarly, the aggregations over different attributes.

2 People can deviate from expected utility for normative ("rational")
 3 reasons, psychological ("irrational") reasons, and tractability reasons. Discrepancies between
 4 expected utility and behavioral decision making need not refute expected utility in a normative
 5 sense. To the contrary, we believe that such discrepancies, and the resolution thereof, are the
 6 essence of expected utility's contribution to improving decision making in the medical and
 7 other areas.

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