

To: Professor John F. Nash

Tilburg, April 1, 1998

Dear Professor Nash,

I would like to ask one question on the history of your famous paper "The Bargaining Problem" in *Econometrica* 18 of 1950. The paper is already famous for what it does on bargaining. It can be famous for another nice idea, "hidden away" at the bottom of page 156. I have enclosed a copy of the relevant (the first three) pages of your paper, plus a paper by Peter Fishburn and me in *Management Science* on the history of the idea discussed hereafter.

The nice idea concerns the first two lines of point 5 in your paper, bottom of p. 156 (p is probability and A, B, and C are lotteries over prizes):

5. If $0 \leq p \leq 1$ and A and B are equally desirable, then $pA + (1-p)C$ and $pB + (1-p)C$ are equally desirable.

Mainly that condition implies that the individual follows expected utility. It is often called "independence" today. The condition was used implicitly, but never stated, by von Neumann and Morgenstern, and it was an omission that they did not state it. People at RAND and elsewhere started discussing it after, e.g., Arrow, Dalkey, Marschak, Rudin, Samuelson. The cited condition may seem simple or trivial, too trivial to bother about where one got it from, but it has nowadays become the center of decision under risk and hundreds of papers discuss it and vary on it, so its origin is highly

important. Marschak and Samuelson published it, Dalkey was the first to point out that this was what von Neumann & Morgenstern had used but not stated. Samuelson wrote a letter to Fishburn and me pointing out that he got the idea from Marschak, and that therefore Marschak was to be credited. Marschak published his work in *Econometrica* 18, 1950, in the same issue as your paper. So, all traces have been led back to Marschak, with one remaining exception: Your paper! Maybe you invented it independently!

Hence my question to you is whether you remember how you got the idea of condition 5 and that it implies expected utility. Your paper states the result as more or less self-evident at the top of page 157. So, do you remember interactions with others that led you or others to know this, or did you find it independently?

I know that such a question, on a seemingly trivial condition, 48 years after publication, may seem silly, but the condition is important for decision under risk so I hope you are willing to let me know what you remember.

Sincerely, Peter Wakker

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To: Professor Peter Fishburn

Tilburg, April 3, 1998

Dear Peter,

This letter describes a meeting I had with John Nash. The meeting completes a "left-over" of our research on the history of vNM-independence, therefore it was for me a dream come true! When we worked on that history, we found that independence emerged from discussions at RAND and elsewhere between Arrow, Dalkey, Marschak, Rudin, and others. All these traces could be reduced to the first written publication, Marschak (1950, *Econometrica*). In particular, Samuelson wrote to us that he got the idea from Marschak. We've corresponded/communicated with all these people and found out as much as could.

One exception remains. There was the Nash (1950, *Econometrica*, "The Bargaining Problem") paper, in the same issue as Marschak's paper, separated by 13 pages, that also stated the independence condition. As far as we could tell, Nash's account was completely independent of all the rest. It would of course be most desirable to communicate with Nash on his memories etc. However, in the days when we did our research, it was said that Nash was in a bad mental state and sensible communication with him was not possible. Now, however, Nash is in better shape. He has spoken at game theory conferences, himself went to get his Nobel prize, in short, he is doing better. Today, April 3, he visited Tilburg University and even gave a lecture there.

I had been excited for many days already, primarily because I did not know in what shape of communication he would be. Well, that was just fine. He was open and not at all distant-like, I had lunch with him and spoke quite a bit with him. A dream come true! One thing he told was that he, being at Princeton, had three times spent a summer at RAND. It was not so easy to make him understand very precisely what I wanted to know. First I showed him his 1950 paper with the relevant lines. He did not remember very well (48 years later that should be no surprise) even the condition. But I told him a bit in great lines what it was about. He thought he had done no more than restate von Neumann-Morgenstern, said that the only thing about axioms is logic, to show they imply what they should. In the afternoon there were lectures to his honor and I spoke about the independence condition history, referring to the joint work with you, and telling that Nash was one of the two inventors of the most important condition in expected utility, that almost nobody was aware of that, including Nash himself who had neither been aware of his contribution there. This was all with him in the audience.

After that I gave him a copy of his 1950 paper with the relevant parts underlined, I gave him a copy of our history paper with relevant parts underlined, and I

added a letter in which I ask him in writing what he remembers of interactions between him and the others concerning independence (copy added). He had no time during the rest of the day (lectures, dinner, etc.) to read the letter but he did immediately have a look at it and seemed interested.

In a break he and I and Hans Peters (a specialist in game theory) spoke about his bargaining game theory paper (the one relevant for us), where outcomes are translated into utility. Nash said: "In those days, utility was already well-established." I could not resist pointing out that he was in fact one of the first to write it correctly.

In general he seemed a modest person who does not want to get too much credit. When we once spoke about something in game theory (I think it was the Nash bargaining solution) he himself brought up that Zeuthen had before done similar things.

So, for now the case is that he does not seem to remember much about the independence condition, thinks he only rewrote von Neumann & Morgenstern. It may well be that he just understood what they wanted and then himself realized what must be needed for it, he was the kind of man who could invent that and find it trivial. It may also be that he heard it from someone other, him being at RAND some times. We cannot know. At any rate, I felt it as an unfinished part of our historical work, now we have done all that can be done. Let us hope that, if memories return to Nash, he will let us know.

Regards, Peter Wakker

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