Chapter 4 bis

EXERCISE 4.5.1.

a) The indifference (*) implies BU(0.9:4; 0.1:0) = BU(1). Because BU(1) = w(1)*U(1) = U(1) = 1, we get

$w(0.9)*U(4) + (1-w(0.9))*U(0) = U(1);$

$w(0.9)*2 = 1;$

$w(0.9) = 0.5.$

BU(1/4) = U(1/4) = 0.5. BU(0.9:1; 0.1:0) = 0.5*1 = 0.5. The agent is indifferent: (0.9:1; 0.1:0) ~ 1/4.

b) BU´(0.9:4; 0.1:0) < BU´(1) = 1. **w´(0.9)***2 + (1–**w´(0.9)**)0 < 1. **w´(0.9)** < ½ = w(0.9). The only thing changed for the calculations here for Keith relative to Mick is that w(0.9) was replaced by the bold **w´(0.9)**, leading to a smaller weight for the highest utility U(4) = 2 in the convex combination of the utilities U(4) and U(0) = 0 when evaluationg the lottery. You can guess this without seeing the algebra if you know that lowering w enhances risk aversion. □

EXERCISE 4.5.2. Consider $x = (p{:}x_1, 1-p{:}x_2)$ with $x_1 \geq x_2$.

$$DA(x) = \frac{pU(x_1) + (1+\beta)(1-p)U(x_2)}{p + (1+\beta)(1-p)} = \frac{p}{p + (1+\beta)(1-p)}U(x_1) + \frac{(1+\beta)(1-p)}{p + (1+\beta)(1-p)}U(x_2).$$

Define $w(p) = \frac{p}{p + (1+\beta)(1-p)}$. Then $DA(x) = w(p)U(x_1) + (1-w(p))U(x_2)$. The function w satisfies all requirements of a probability weighting function, so we have biseparable utility. □

Chapter 5 bis

EXERCISE 5.2.1.

(a) DU = 9.86 and PV = 97.19.

(b)

(0.5:100): DU = 9.86; PV = 97.19

(0.5+1:105): DU = 9.92; PV = 98.44

The late large payment (0.5+1:105) is preferred, suggesting decreasing impatience.

(c) (2: 105) has DU = 9.86; PV = 97.19. So the two are indifferent.

(d) DU = 9.88 and PV = 97.59.

(e)

(0.5:100): DU = 9.88; PV = 97.59.

(0.5+1:103.943): DU = 9.95; PV = 98.99

The late large payment (0.5+1:103.943) is preferred, suggesting decreasing impatience.

(f) (2.5: 103.943) has DU = 9.88; PV = 97.59. So the two are indifferent.

(g) For a = 1 the extra time the person is willing to wait after the 0.5 delay has increased by ε = 0.5 (from 1.5 to 2), but for a = 2 it has increased by ε = 1 (from 1.5 to 2.5). This suggests that impatience decreased more for a = 2 than for a = 1. □

EXERCISE 5.3.1.

a) Take any indifference

$(0{:}\sigma) \sim (\ell{:}\lambda)$

with $\ell > 0$, $\lambda > \sigma > 0$. By continuity and strict increasingness of utility, you can always take an arbitrary $\lambda > 0$ and then find the corresponding σ. By discounted utility we have

$$U(\sigma) = \beta\delta^{\ell}U(\tau) \qquad (*)$$

Take an arbitrary delay $d > 0$, and consider the preference between $(d{:}\sigma)$ and $(d+\ell{:}\lambda)$. They have discounted utility (DU) values $\beta\delta^{d}U(\sigma)$ and $\beta\delta^{d+\ell}U(\lambda)$. Multiplying Eq. * by δ^{d} gives $\delta^{d}U(\sigma) = \beta\delta^{d+\ell}U(\tau)$. It implies $\beta\delta^{d}U(\sigma) < \beta\delta^{d+\ell}U(\tau)$. Thus

$(d{:}\sigma) \prec (d+\ell{:}\lambda)$.

The two displayed preferences give a violation of stationarity.

b) If timepoint 0 is not involved, then quasi-hyperbolic discounting is equivalent to exponential discounting multiplied by a positive β, which gives the same preferences as with constant discounting. Then stationarity cannot be violated. □

EXERCISE 5.3.2.

a) $\varepsilon = 0$.

b) $DU(0.5{:}100) = 0.95 \times e^{-0.01\times 0.5} \times \sqrt{100} = 0.95 \times 0.995 \times 10 = \mathbf{9.45}$.

$PV(0.5{:}100) = U^{-1}(9.45) = 9.45^2 = \mathbf{89.35}$.

c) (0.5:100): DU = 9.45 (and PV = 89.35).

$DU(1.5{:}113.042) = 0.95 \times e^{-0.01\times 1.5} \times \sqrt{113.042} = 0.95 \times 0.9851 \times 10.632 = \mathbf{9.95}$.

$\big(PV(0.5{:}100) = 9.95^2 = \mathbf{99.01}\big)$. DU(1.5:113.042) > DU(0.5:100), implying (1.5:113.042) $\succ$ (0.5:100).

We have (0:100) ~ (1:113.042) and (0.5:100) $\prec$ (1.5:113.042), suggesting **decreasing impatience**.

d) $DU(6.6295{:}\ 113.042) = 0.95 \times e^{-0.01\times 6.6295} \times \sqrt{113.042} = 0.95 \times 0.9358 \times 10.632 =$ **9.45**. ($PV(6.6295{:}\ 113.042) = 9.45^2 = \mathbf{89.35}$.) The DU is by two digits equal to DU(0.5:100). (Likewise, PVs are equal by two digits.) So the two are indifferent: (6.6295: 113.042) ~ (0.5:100).

e) $DU(0.5{:}100) = 0.90 \times e^{-0.01\times 0.5} \times \sqrt{100} = 0.90 \times 0.995 \times 10 = \mathbf{8.96}$.

$PV(0.5{:}100) = 8.96^2 = \mathbf{80.19}$.

f) (0.5:100): DU = 8.96; PV = 80.19.

$DU(1.5{:}125.95) = 0.90 \times e^{-0.01\times 1.5} \times \sqrt{125.95} = 0.90 \times 0.985 \times 11.22 = \mathbf{9.95}$.

$\big(PV(0.5{:}100) = 9.95^2 = \mathbf{99.00}\big)$. DU(1.5:125.95) > DU(0.5:100), implying (1.5:125.95) $\succ$ (0.5:100).

We have (0:100) ~ (1:125.95) and (0.5:100) $\prec$ (1.5:125.95), suggesting **decreasing impatience**.

g) $DU(12.04{:}\ 125.95) = 0.90 \times e^{-0.01\times 12.04} \times \sqrt{125.95} = 0.90 \times 0.886 \times 11.22 = \mathbf{8.96}$.

($PV(12.04{:}\ 125.95) = 8.96^2 = \mathbf{80.19}$.) The DU is by two digits equal to DU(0.5:100). (Likewise, PVs are equal by two digits.) So the two are **indifferent**: (12.04: 125.95) ~ DU(0.5:100).

h) For $\delta = 0.95$ the *extra* time the person is willing to wait after the 0.5 delay has increased by $\varepsilon = 5.1295$ (from 1.5 to 6.6295), but for $\delta = 0.90$ the *extra* time the person is willing to wait after the 0.5 delay has increased by $\varepsilon = 10.536$ (from 1.5 to 12.036). The second *extra* time 10.536 exceeds the first *extra* time 5.1296. The latter exceeding suggests that impatience has decreased more for $\beta = 0.90$ than for $\beta = 0.95$, so, the *decrease* of impatience has become stronger for $\beta = 0.90$.
Some students only note stronger impatience for $\beta = 0.90$ than for $\beta = 0.95$, i.e. longer willingness to wait, and erroneously do not consider the *decrease of* impatience. □

Chapter 6

EXERCISE 6.1.1.

(a) In y, the expected utility of both agents is $0.5 \times 4 + 0.5 \times 0 = 2$. In z, it is $0.5 \times 6 + 0.5 \times 0 = 3$. Hence, in z both agents are better off. By Pareto optimality, z is preferred by the social planner.

(b) I only give formulas for agent 1, because things are the same for agent 2. For agent 1, the general evaluation of (x_1,x_2) (meaning utility $u_1 = x_1$ and $u_2 = x_2$) is

$$x_1 - b \times \max\{x_2 - x_1, 0\}$$

with $b \geq 0$.
For prospect y, (4, 4) has value 4 for agent 1 and (0, 0) has value 0 for agent 1. The resulting expected utility is $0.5 \times 4 + 0.5 \times 0 = 2$.
Now we turn to prospect z.
(6, 0) has value 6 for agent 1 (no negative utility from agent 2 being poorer).
(0, 6) has value $-b \times 6$ for agent 1.
The expected utility is $3 - 3b$ under z. This is strictly below 2, the expected utility under y, whenever $b > 1/3$. Then agent 1, and agent 2 similarly, prefer y to z, and so does the social planner. So $b > 1/3$ is necessary and sufficient to get $y \succ z$. □

Chapter 7

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Chapter 8

EXERCISE 8.1.1.

(a) (0.5:T, 0.5:B) strictly dominates M. Hence M can be deleted. After this deletion, L is strictly dominated by R and can be deleted. After this, T strictly dominates B and we delete B. (T,R) remains as the unique Nash equilibrium, yielding outcome 7 [2].

(b) NAU if L is chosen:

NAU of T: the minimal possible outcome of T is 4, and its expected utility is 4 too, given that P(L) = 1. Hence NAU of T is $0.75 \times 4 + 0.25 \times 4 = 4$.

Value M = 5 (it is $0.75 \times 5 + 0.25 \times 5 = 5$, but it is immediately obvious that it can only be 5).

Value of B: the minimal possible value of B is 4 (this is considered possible and counted as minimal, even if P(L) = 1.), and its expected utility is 7. B's NAU is $0.75 \times 4 + 0.25 \times 7 = 4.75$.

NAUs if R is chosen:

NAU of T = $0.75 \times 4 + 0.25 \times 7 = 4.75$.

NAU of M = 5.

NAU of B = $0.75 \times 4 + 0.25 \times 4 = 4$.

Both if L is chosen, and if R is chosen, M has a strictly higher value than both T and B.

(c) From (b) is follows that M is chosen for sure. Then the value of L is 0.75×1 + 0.25×2 = 1.25. The value of R is 0.75×1 + 0.25×1 = 1. Hence L is chosen. Thus we get the equilibrium (M,L). □

Chapter 9

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Chapter 10

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Chapter 11

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Final Conclusion

EXERCISE 12.1.1. Here is how this exercise was graded in an exam: Some students just listed general biases or general theoretical ideas. This is bad, for one reason because it is not what is asked. Several students only listed some biased taught, and simply said they will not do it. This is simple and uninformative/uncreative. Several students mentioned concrete events such as recent decisions in saving, insurance, sales, and so on, and how they did experience, or could experience new insights due to behavioral theories. Many mentioned self-control problems as with time inconsistency. This is good. Some students further expressed original/personal opinions/doubts on rationality in their decisions, and this can be very good. □