Exercises Behavioral Economics

by Peter P. Wakker

Many exercises below are not part of the homework. They are for volunteers (and other teachers). For them, no solutions are provided.

Chapter 1

EXERCISE 1.1.1. What is your solution to the water-diamond paradox?

Some more details: Your water consumption this year serves for you to survive this year. You would use a diamond only to be rich instead of poor. This suggests that the utility of the water exceeds that of the diamond. But you would pay more for the diamond than for the water. This suggests that the utility of the diamond exceeds that of the water. Which of these two suggestions is correct in your case, based on revealed preference? Can you imagine a change in your current situation that would reverse what is the correct suggestion conclusion?

EXERCISE 1.2.1. Assume a nonempty set X, with elements called *prospects* and denoted x, y, z, etc. By C we denote a choice function on X. That is, to every finite subset A of X, C assigns a nonempty subset $C(A) \subset A$. Of course, because A is finite, so is $C(A)$. By R, P, I we denote revealed preference relations on X. That is, they are binary relations on X, defined by:

xRy if there exists a finite subset $A \subset X$ with $x \in C(A), y \in A$ (*revealed weak preference*);

xPy if there exists a finite subset $A \subset X$ with $x \in C(A), y \in A$, $y \notin C(A)$ (*revealed strict preference*);

xIy if there exists a finite subset $A \subset X$ with $x \in C(A), y \in C(A)$ (revealed indifference).

By definition, C *maximizes a weak order* $\succcurlyeq$ if $\succcurlyeq$ is a weak order on X (i.e., it is transitive and complete) and

$C(A) = \{x \in A: x \succcurlyeq y \text{ for all } y \in A\}$

for every finite $A \subset X$.

Given $\succcurlyeq$, the notation $\succ$, ~, $\preccurlyeq$, and $\prec$ is as usual:

$x \succ y$ if $x \succcurlyeq y$ and not $y \succcurlyeq x$;

$x \sim y$ if $x \succcurlyeq y$ and $y \succcurlyeq x$;

$x \preccurlyeq y$ if $y \succcurlyeq x$;

$x \prec y$ if $x \succ y$.

C satisfies the *weak axiom of revealed preference* (*WARP*) if for no x, y we have xRy and yPx.

(a) Assume that C maximizes a weak order $\succcurlyeq$. Show that

$x \succcurlyeq y$ implies xRy.

(b) Assume that C maximizes a weak order $\succcurlyeq$. Show that

xRy implies $x \succcurlyeq y$.

(c) Assume that C maximizes a weak order $\succcurlyeq$. Show that

$x \succ y$ implies xPy.

(d) Assume that C maximizes a weak order $\succcurlyeq$. Show that

xPy implies $x \succ y$.

(c) Assume that C maximizes a weak order $\succcurlyeq$. Show that
$x \sim y$ implies xIy.

(f) Assume that C maximizes a weak order $\succcurlyeq$. Show that
xIy implies $x \sim y$.

(g) Assume that C maximizes a weak order $\succcurlyeq$. Show that C satisfies WARP.

(h) Assume that C satisfies WARP. Show that C maximizes a weak order $\succcurlyeq$.[1]

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[1] This part is considerably more difficult than the other parts.

EXERCISE 1.3.1. Assume that prospect a is chosen from $\{a, b, c\}$, and c from $\{b, c\}$. Specify a utility function U corresponding with these choices. What is chosen from $\{a, c\}$?

EXERCISE 1.4.1. Consider the prospect $(0.99: 4.01, 0.01: 3.01)$, yielding €4.01 with probability 0.99 and €3.01 with probability 0.01. Its expected value is 4. Assume utility $U(\alpha) = \alpha^2$. Assume expected utility. What is the expected utility of the prospect? What is preferred, the prospect or receiving €4 for sure? Given that U is convex, we have risk seeking under expected utility. Verify that this is consistent with your answer.

EXERCISE 1.4.2. Assume EU with $U(\alpha) = \alpha^{0.2075}$. Calculate the EUs (expected utilities) and the CEs (certainty equivalents)[2] of the following prospects:

a) (0.80:200; 0.20:39)
b) (0.80:400; 0.20:78)
c) (0.80:2000; 0.20:390)

When comparing the three CEs, do you notice something remarkable?

EXERCISE 1.4.3. Assume EU with $U(\alpha) = 1 - \exp(-0.001 \times \alpha)$. Calculate the EUs and the CEs of the following prospects:

a) (0.80:230; 0.20:90)
b) (0.80:330; 0.20:190)
c) (0.80:1230; 0.20:1090)

When comparing the three CEs, do you notice something remarkable?

EXERCISE 1.4.4 [Insurance exercise]. An insurance company insures bikes of value €200 for one year. These bikes have a probability $p = 0.05$ of being stolen during one

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[2] The money amount α is the *certainty equivalent* of a prospect x if $\alpha \sim x$.

year, both in Rotterdam and in Amsterdam. The prospect faced (throughout taken after a year) is therefore (0.05: –200, 0.95:0). The expected loss, denoted EV, is –10. CE denotes the certainty equivalent of the prospect for some client, i.e. CE ~ (0.05: –200, 0.95:0). Clients are risk averse and, hence, CE < –10. The insurance company wonders if it can make more money from an average client in Amsterdam or in Rotterdam. Define the *risk premium* for a client as EV – CE. By risk aversion, the premium is positive.

a) Explain that the client is willing to pay an insurance premium up to –CE for insurance.
b) Explain that the risk premium is the maximal profit that the insurance company can make per average client, assuming that it will have many average clients.
c) Now assume that the average client in Rotterdam and in Amsterdam are both rational and maximize EU. The average Rotterdam client has $U(\alpha) = (3000+\alpha)^{0.5}$, and the average client in Amsterdam has $U(\alpha) = (3000+\alpha)^{0.2075}$. How much money can the insurance company make in Amsterdam, and how much in Rotterdam, per average client? Where will it rather go, assuming the same number of clients in both cities?

EXERCISE 1.5.1. Consider a state space $\{s_1,s_2,s_3\}$. Assume EU with the normalized $U(0) = 0$ and $U(100) = 1$, and assume

$(s_1{:}25, s_2{:}0, s_3{:}0) \sim (s_1{:}0, s_2{:}25, s_3{:}0) \sim (s_1{:}0, s_2{:}0, s_3{:}25)$ and

$(s_1{:}25, s_2{:}25, s_3{:}25) \sim (s_1{:}100, s_2{:}0, s_3{:}0)$.

What are $P(E_1)$, $P(E_2)$, $P(E_3)$, and $U(25)$?

EXERCISE 1.5.2. Consider a state space $\{s_1,..., s_6\}$.

When choosing between the three acts

$(s_1{:}100, s_2{:}100, s_3{:}100, s_4{:}100, s_5{:}100, s_6{:}800)$

$(s_1{:}100, s_2{:}100, s_3{:}200, s_4{:}200, s_5{:}300, s_6{:}300)$

$(s_1{:}200, s_2{:}200, s_3{:}300, s_4{:}300, s_5{:}100, s_6{:}100)$

the lower one is strictly preferred because in most states, four out of six, it is best.

When choosing between the three acts

(s_1:100, s_2:100, s_3:100, s_4:800, s_5:100, s_6:100)
(s_1:200, s_2:200, s_3:300, s_4:300, s_5:100, s_6:100)
(s_1:300, s_2:300, s_3:100, s_4:100, s_5:200, s_6:200)
the lower one is strictly preferred because in most states, four out of six, it is best.
When choosing between the three acts
(s_1:100, s_2:800, s_3:100, s_4:100, s_5:100, s_6:100)
(s_1:300, s_2:300, s_3:100, s_4:100, s_5:200, s_6:200)
(s_1:100, s_2:100, s_3:200, s_4:200, s_5:300, s_6:300)
the lower one is strictly preferred because in most states, four out of six, it is best.
Can these preferences be modeled using subjective expected utility? □

EXERCISE 1.6.1. The Theorem of Koopmans (1968) of Wakker's slides (§1.6) claims that constant discounted utility implies additive separability and stationarity. Prove this.

EXERCISE 1.6.2. When choosing between the two income streams
(100, 100, 200, 200, 300, 300, 0, 0, …..)
(200, 200, 300, 300, 100, 100, 0, 0, …..)
the lower one is strictly preferred because in most of the relevant timepoints, four out of six, it is best.
When choosing between the two income streams
(200, 200, 300, 300, 100, 100, 0, 0, …..)
(300, 300, 100, 100, 200, 200, 0, 0, …..)
the lower one is strictly preferred because in most of the relevant timepoints, four out of six, it is best.
When choosing between the two income streams
(300, 300, 100, 100, 200, 200, 0, 0, …..)
(100, 100, 200, 200, 300, 300, 0, 0, …..)
the lower one is strictly preferred because in most of the relevant timepoints, four out of six, it is best.
Can these preferences be modeled using discounted utility (DU)? □

EXERCISE 1.6.3. Prospects are income streams $x = (x_0,x_1,x_2)$, yielding x_0 now, x_1 in one month, and x_2 in two months. So, we consider only three timepoints. An agent evaluates income streams through a function $V(x_0,x_1,x_2)$ in two stages.

STAGE 1. She calculates discounted utility with $U(\alpha) = \alpha$ and $\delta = 1$. That is, she just takes the sum of incomes, $x_0 + x_1 + x_2$.

STAGE 2. The agent dislikes decreases in income. For each j with $x_j < x_{j-1}$, she subtracts $(x_{j-1} - x_j)/2$ from the sum of stage 1.

For example,

$V(1,3,7) = 1 + 3 + 7 - 0 = 11.$

$V(1,7,3) = 1 + 7 + 3 - (7-3)/2 = 11-2 = 9.$

$V(7,3,1) = 7 + 3 + 1 - (7-3)/2 - (3-1)/2 = 11-2-1 = 8.$

Does the agent satisfy additive separability? If you cannot find the full solution, you can speculate intuitively. □

EXERCISE 1.7.1. Assume a society consisting of three agents, $j = 1,2,3$ with preference relations $\succcurlyeq_j$ over probability distributions over social states, and a social planner with preference relation $\succcurlyeq$ over probability distributions over social states. All the conditions of the Theorem of Harsanyi (1955) of Wakker's slides (§1.7) hold. That is, every agent j maximizes expected utility (EU) with utility function $u_j(x)$ and the social planner maximizes EU with $U(x) = \sum_{j=1}^{3} a_j u_j(x)$ for some a_j, with $a_j \geq 0$ for every j.

Everyone agrees that w is the worst social state and b is the best social state, and we assume $u_j(w) = 0$ and $u_j(b) = 1$ for all j. We also assume $U(w) = 0$ and $U(b) = 1$. Hence, all u_j and U values are between 0 and 1.

a) Prove $\sum_{j=1}^{3} a_j = 1$.

b) In addition to the assumptions made, assume three social states x^1 (agent 1 is king), x^2 (agent 2 is king), and x^3 (agent 3 is king). The three agents disagree much on the desirability of these three states. Each agent j prefers x^j most and considers it equally good as b, but disprefers x^i for $i \neq j$ much and considers it equally bad as w. That is:

$x^1 \sim_1 b$, $x^2 \sim_1 w$, $x^3 \sim_1 w$,

$x^1 \sim_2 w$, $x^2 \sim_2 b$, $x^3 \sim_2 w$,

$x^1 \sim_3 w$, $x^2 \sim_3 w$, $x^3 \sim_3 b$.

The social planner is indifferent between the three social states:

$x^1 \sim x^2 \sim x^3$.

What are a_1, a_2, and a_3?

c) In addition to the assumptions made above including those of part b), assume a fourth social state y. The three agents agree on the desirability of y in the sense that there exists a $0 < p < 1$ such that

$y \sim_j (p{:}b, 1-p{:}w)$ for all j.

The social planner is indifferent between all four social states:

$x^1 \sim x^2 \sim x^3 \sim y$.

What are $u_1(y)$, $u_2(y)$, $u_3(y)$, $U(y)$, and p?

Chapter 2

EXERCISE 2.2.1 [Co-existence of gambling and insurance violates EU].
Concavity of a function U is defined by the requirement

For all $\gamma > \beta$, and $0 < \lambda < 1$: $U(\lambda\gamma + (1-\lambda)\beta) \leq \lambda U(\gamma) + (1-\lambda)U(\beta)$. (1)

A sufficient condition for concavity, assuming continuity of U, is the weaker

For all $\gamma > \beta$, there exists $0 < \lambda_{\gamma,\beta} < 1$ such that:

$U(\lambda_{\gamma,\beta}\gamma + (1-\lambda_{\gamma,\beta})\beta) \;\leq\; \lambda_{\gamma,\beta}U(\gamma) + (1-\lambda_{\gamma,\beta})U(\beta)$ (2)

(Hardy, Littlewood & Pòlya (1934, Observation 88 in §3.7).
Assume that an agent is prone to insurance in the sense that

$(p{:}\gamma, 1-p{:}\beta) \prec p\gamma + (1-p)\beta$ for all $\gamma > \beta$ and $0.2 < p < 1$. (3)

Assume that the agent likes to gamble in having the preference

$(0.00001{:}10^6, 0.99999{:}0) \succ 10$. (4)

Is it possible that the agent maximizes EU with a continuous strictly increasing utility function U?

EXERCISE 2.3.1 [Ellsberg paradox violates probabilistic sophistication]. Probabilistic sophistication generalizes EU as follows. To prepare, consider the subjective expected utility model of Savage (1954). Subjective probabilities $p_1,\ldots,p_n$ are assigned to the states $s_1,\ldots,s_n$. The prospect illustrated there is mapped into the probability distribution $(p_1{:}x_1,\ldots,p_n{:}x_n)$. V denotes a general function assigning a real number to each probability distribution. It may be different than the EU function.[3] It can be a different kind of function. We assume that V satisfies strict stochastic dominance: moving positive probability mass from an outcome to a strictly higher outcome strictly increases the V value. Preferences over prospects maximize V. Prove that the Ellsberg paradox falsifies not only Savage's subjective expected utility, but also probabilistic sophistication.

EXERCISE 2.4.1 [Intertemporal choice]. An agent, who will live infinitely many years, has to do a job for her tax declarations in one of the coming seven years. The preferences of the agent in year t are denoted $\succcurlyeq_t$. That is, these are the preferences when the decisions are made in year t. Thus, $\succcurlyeq_0$ denotes preferences regarding decisions made in $t = 0$ (this year), $\succcurlyeq_1$ denotes preferences regarding decisions made next year, and $\succcurlyeq_6$ denotes preferences regarding decisions made in the last of the seven years.

The preference relations $\succcurlyeq_t$ all satisfy transitivity (implying transitivity of strict preference: $a \succ_t b$ & $b \succ_t c \Rightarrow a \succ_t c$) and completeness ($a \succcurlyeq_t b$ or $b \succcurlyeq_t a$ for all a,b). These conditions preclude [$a \succ_t b$ and $b \succ_t a$] (Micro I).

The job has to be done either in year $t = 0$, or in $t = 1$, or …, or in $t = 6$. It costs c (so $c < 0$) at the moment when done but delivers reward r (so $r > 0$) the year after it was done. It has to be done exactly once. In all years not affected by the decision, the payoff is 0. Thus, for instance, doing the job in year $t = 2$ gives income stream (0,0,c,r,0,0,0,0,0,0,…). As another example (0,0,0,0,c,r,0, 0, …) means income 0 in years $t = 0$, $t = 1$, $t = 2$, and $t = 3$, income c in year $t = 4$ (remember $c < 0$), income r in year $t = 5$, and income 0 in all years that follow. It results from doing the job in year $t = 4$.

[3] For instance, it may be a mean-variance function as widely used in finance.

This year ($t = 0$) the agent faces the decision of doing the job this year or not. If she does not do the job this year, then she faces the same decision again next year. She faces the same decision every year until the job is done, or until it is year $t = 6$ when the job just has to be done if it had not been done before.

This year the agent has the following preferences:

(doing the job next year) $\succ_0$

(doing the job this year) $\succ_0$

(doing the job in year $t = 6$).

That is, for strict preferences $\succ_0$ this year we have (where in all years not affected by the decision, the payoff is 0):

$(0,c,r,0,0,0,0,0,0,\ldots) \succ_0$

$(c,r,0,0,0,0,0,0,0,\ldots) \succ_0$

$(0,0,0,0,0,0,c,r,0,\ldots)$.

Note how denoting the sequences below one another conveniently displays which payments are in common years.

(a) Show that the agent cannot satisfy stationarity.

(b) Speculate on when the agent will do the job if she is naïve.

(c) Speculate on when the agent will do the job if she is sophisticated.

A useful notation is to write c_j for doing the job on day j.

EXERCISE 2.5.1. Assume that an agent is evaluating future jobs where only two attributes are relevant: (1) Net salary per month. (2) Number of vacation days per year. The agent prefers the more salary the better, and the more vacation days the better. We ask the agent:

QUESTION 1. Consider (€X, 20 days) versus (€3000, 10 days). Which salary X makes you indifferent?

The person gives an answer that we denote Y. Some time later, when the person forgot that we asked this question (which works well in experiments with many questions), we ask

QUESTION 2. Consider (€Y, 20 days) versus (€3000, Z days). How many vacation days Z make you indifferent?

a) What answer Z will homo economicus give?
b) Now assume that the subject is homo sapiens, and is subject to scale compatibility. Speculate on whether we will find Z > 10, Z = 10, or Z < 10. □

EXERCISE 2.5.2. Consider the following choices and preferences. Indicate, for the preferences in each of the six parts, the preference conditions that are violated there, if any, and/or which biases are effective, if any. Throughout, outcome M denotes million euro and outcome 0 denotes 0 euro, with the usual notation for lotteries.(a)

(0.90: €0, 0.06: €45, 0.01: €30, 0.01: €–15, 0.02: €–15)

$\succ$

(0.90: €0, 0.06: €45, 0.01: €45, 0.01: €–10, 0.02: €–15).

(b)

(0.80: 4M, 0.20: 0) $\prec$ 1M

&

(0.20: 4M, 0.80: 0) $\succ$ (0.25: 1M, 0.75: 0)

These four lotteries can be denoted L_1, L_2, L_3, and L_4, respectively.

(c)

1M $\succcurlyeq$ (0.10:40M, 0.90:0)

&

(0.001:40M, 0.99:1M, 0.009:0) $\succ$ 1M

The first three lotteries can be denoted L_1, L_2, and L_3, respectively.

(d)

(0.90: €0, 0.06: €45, 0.01: €30, 0.03: €–15)

$\succ$

(0.90: €0, 0.07: €45, 0.01: €–10, 0.02: €–15)

(e)

(0.97: €4, 0.03: €0) $\succ$ (0.31: €16, 0.69: €0)

&

€4.25 $\sim$ (0.31: €16, 0.69: €0)

&

$$€3.95 \sim (0.97{:}\ €4,\ 0.03{:}\ €0)$$

(f)

$$(0.01{:}\ 40\text{M},\ 0.10{:}\ 0,\ 0.89{:}\ 0) \succ (0.01{:}\ 10\text{M},\ 0.10{:}\ 10\text{M},\ 0.89{:}\ 0)$$

&

$$(0.01{:}\ 40\text{M},\ 0.10{:}\ 0,\ 0.89{:}\ 10\text{M}) \prec (0.01{:}\ 10\text{M},\ 0.10{:}\ 10\text{M},\ 0.89{:}\ 10\text{M}) \quad \square$$

Chapter 3

EXERCISE 3.1. Read the paper

https://doi.org/10.2307/1914185

Kahneman, Daniel & Amos Tversky (1979) "Prospect Theory: An Analysis of Decision under Risk," *Econometrica* 47, 263–291

except its appendices. The paper introduced a new theory for decision under risk, called prospect theory, extending expected utility. However, the paper had much broader implications, affecting all of economics. It was the first to produce a theory explicitly meant to capture emotions beyond rationality, but yet allowing for quantitative measurements and hard predictions. This had been thought to be impossible up to that point. Thus, the paper gave birth to what is nowadays called behavioral economics. It is the most cited paper ever published in an economic journal.

Write ¾-1 page (250-350 words) describing, among others:

(1) What do you think are the bottom-line findings/conclusions/take-aways of the paper?

(2) What did you like, and what not? Try to mention at least one point you liked, and one point you did not like, about the paper. Relatedly:

(3) What is interesting (mention at least one point)? What are criticisms? Can be combined with the preceding point.

In (2) & (3), please do NOT discuss the hypothetical nature of the choices observed.

(4) Add a word count, i.e., write at the bottom of your report how many words it took.

EXERCISE.

E 3.2. Read the following paper.

https://doi.org/10.1257/000282803321947001

Thaler, Richard H. & Cass R. Sunstein (2003) "Libertarian Paternalism," *American Economic Review, Papers and Proceedings* 93, 175–179.

Read the paper, and make up your mind on the following questions.

(1) What do you think are the bottom-line findings/conclusions/take-aways of the paper?
(2) What did you like, and what not? Try to mention at least one point you liked, and one point you did not like, about the paper. Relatedly:
(3) What is interesting (mention at least one point)? What are criticisms? Can be joint with the preceding point.

An objection that people may raise against behavioral economics is that it is paternalistic to think that you, with rational economic theories, can get better decisions. This paper illustrates that strict paternalism can often be avoided, while still behavioral economics is useful.

C1. P. 175, 2nd column, last para, middle: welfare is something like happiness.

C2. P. 175, 1st column, 1st para, ¼, sentence describes part of the historical line of this course: "Research by psychologists and economists over the past three decades has raised questions about the rationality of the judgments and decisions that individuals make."

Q.3. p. 176 bottom and p. 177 bottom: Do you know a case in your mother country where the choice of a default option was debated?

Q.4. p. 178: Why do the authors think that joining the savings plan is better?

Chapter 4

EXERCISE 4.1.1. Assume OPT (as defined in this course in the powerpoint slides, and slightly deviating from Kahneman & Tversky 1979), and assume that w is nonlinear. Then there exist two probabilities p_1 and p_2 such that $w(p_1 + p_2) \neq w(p_1) + w(p_2)$. You may assume this and, further, that $p_1 + p_2 < 1$. Assume that w and U are continuous. Use two prospects $(p_1: \alpha+\varepsilon, p_2{:}\alpha, p_3{:}0)$ and $(p_1+p_2{:}\alpha, p_3{:}0)$ with $\alpha > 0$ to show that stochastic dominance is violated. Stochastic dominance requires that improving an outcome leads to a preferred prospect, and worsening an outcome leads to a less preferred prospect.

EXERCISE 4.2.1. We consider decision under risk, and only consider gains $\alpha \geq 0$, for an agent John with preference relation $\succcurlyeq$. Assume the following indifference of John:

$$(0.9{:}4;\ 0.1{:}0) \ \sim\ 1. \qquad (*)$$

Throughout, for all theories considered, assume that John's utility function U is the squareroot function: $U(\alpha) = \sqrt{\alpha}$.

(<u>a</u>) Given the function U as specified, does expected utility (EU) hold for John?

(b) Assume Chew's weighted utility WU for John, deviating from EU by the additional function f. Assume $f(0) = 1$. What is f(4)? Assume further:

$$f(\alpha) = \frac{1}{\lambda\alpha+1}$$

for a constant $\lambda > 0$. What is the preference between $(0.9{:}1;\ 0.1{:}0)$ and 1/4?

(c) Now consider Paul, another agent, with preference relation $\succcurlyeq'$. He does not have the indifference in (*). Paul has $(0.9{:}4;\ 0.1{:}0) \prec' 1$. Assume that Paul satisfies Chew's weighted utility WU´ with the same utility function[4] $U'(\alpha) =$

[4] We use a prime only to distinguish this agent. Hence, for functions, the prime does not designate a

$\sqrt{\alpha}$ as for John, but with f´ instead of f. Assume f´(0) = f(0) = 1. What do we have, f´(4) < f(4), f´(4) = f(4), or f´(4) > f(4)?

EXERCISE 4.2.2. Assume the same insurance problem as in Exercise 1.4.4, and the same utility functions. Only now the insurance company discovers that people in Amsterdam, unlike in Rotterdam, are not rational, but violate EU. They discover that weighted utility WU describes the preference relation of the average Amsterdam client well, who double focuses on –200. That is, whereas f(0) = 1, we have f(–200) = 2. How much money can the insurance company now make from the average Amsterdam client?

EXERCISE 4.2.3. *Weak betweenness* holds if

$$x \sim y \Rightarrow \lambda x + (1-\lambda)y \sim x$$

for all prospects x, y, and $0 \leq \lambda \leq 1$. If follows from betweenness by applying betweenness twice, first with all preference one way, and then with all preferences reversed. It can be seen that under continuity and stochastic dominance, weak betweenness implies betweeenness, so is logically equivalent. Show that Chew's weighted utility (WU) implies weak betweenness without using the info from the slides that WU implies betweenness.

EXERCISE 4.3.1 (compare Exercise 4.2.1). We consider decision under risk, and only consider gains $\alpha \geq 0$, for an agent George with preference relation $\succcurlyeq$. Assume the following indifference of George:

$$(0.9{:}4;\ 0.1{:}0) \ \sim\ 1. \qquad (*)$$

Assume that George's utility function U is the squareroot function: $U(\alpha) = \sqrt{\alpha}$.

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derivative.

(a) Assume Gul's disappointment aversion theory DU, deviating from EU by the additional parameter $\beta > 0$. What is β? What is the preference between (0.9:1; 0.1:0) and 1/4?

(b) Now consider Ringo, another agent, with preference relation $\succcurlyeq'$. He does not have the indifference in (*). Ringo has (0.9:4; 0.1:0) $\prec'$ 1. Assume that Ringo satisfies Gul's disappointment aversion DU´ with the same utility function[5] $U'(\alpha) = \sqrt{\alpha}$ as for George, but with β' instead of β. What do we have, $\beta' < \beta$, $\beta' = \beta$, or $\beta' > \beta$?

EXERCISE 4.3.2. Assume the same insurance problem as in Exercise 1.4.4, and the same utility functions. Now the insurance company discovers that disappointment aversion theory describes the preference relation of the average Amsterdam client well, who threefold focuses on disappointing outcomes; i.e., $\beta = 2$. How much money can the insurance company now make from the average Amsterdam client?

EXERCISE 4.3.3. Show that Gul's disappointment aversion theory implies weak betweenness (defined in Exercise 4.2.3).

EXERCISE 4.5.1 (compare Exercises 4.2.1 and 4.3.1). We consider decision under risk, and only consider gains $\alpha \geq 0$, for an agent Mick with preference relation $\succcurlyeq$. Assume the following indifference of Mick:

$$(0.9{:}4;\ 0.1{:}0) \sim 1. \qquad (*)$$

Assume that Mick's utility function U is the squareroot function: $U(\alpha) = \sqrt{\alpha}$.

(a) Assume biseparable utility BU for Mick, deviating from EU by the additional function w. What is w(0.9)? What is Mick's preference between (0.9:1; 0.1:0) and 1/4?

[5] Again, we use a prime to distinguish this agent, and the prime does not designate a derivative.

(b) Now consider Keith, another agent, with preference relation $\succcurlyeq'$. He does not have the indifference in (*). Keith has (0.9:4; 0.1:0) $\prec'$ 1. Assume that Keith satisfies biseparable utility BU´ with the same utility function[6] $U'(\alpha) = \sqrt{\alpha}$ as for Keith, but with w´ instead of w. What do we have, $w'(0.9) < w(0.9)$, $w'(0.9) = w(0.9)$, or $w'(0.9) > w(0.9)$?

EXERCISE 4.5.2. Show that on the domain of two-outcome prospects, DA theory is a special case of biseparable utility.

EXERCISE 4.5.3. Assume the same insurance problem as in Exercise 1.4.4, and the same utility functions. Now the insurance company discovers that biseparable utility describes the preference relation of the average Amsterdam client well, with the weighting function in §4.1 of Wakker's powerpoint slides:

$$w(p) = \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\gamma})^{1/\gamma}} \text{ with } \gamma = 0.61.$$

How much money can the insurance company now make from the average Amsterdam client?

EXERCISE 4.6.1. Consider the prospect (0.99:4.01, 0.01:3.01), yielding €4.01 with probability 0.99 and €3.01 with probability 0.01. Its expected value is 4. Assume utility $U(\alpha) = \alpha^2$, so it is the square function. Assume neo-additive utility (NAU), with the same U as above, and pessimism index $\sigma = 0.2$ and optimism index $\tau = 0.1$. What is the NAU of the prospect? What is the NAU of receiving €4 for sure? What is preferred, the prospect or receiving €4 for sure? Does this suggest risk aversion or risk seeking? Compare the result with Exercise 1.4.1.

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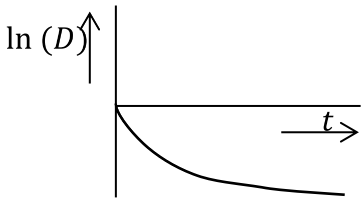
[6] Again, we use a prime to distinguish this agent, and the prime does not designate a derivative.

99% of the economists today think that, also if we do not have the expected utility model, then still convex utility means risk seeking. Are they right? How many generations do you think it will take the economic field to correct this incorrect belief?

Chapter 5

EXERCISE 5.1.1. Assume discounted utility with discount function D and utility function U. Assume that ln (D) is convex, implying

$$\ln\big(D(0)\big) - \ln\big(D(\ell)\big) \geq \ln\big(D(d)\big) - \ln\big(D(d+\ell)\big) \text{ for all } \ell \geq 0, d \geq 0. \quad (*).$$



Show that decreasing impatience holds in the sense that

$$(0:\sigma) \sim (\ell:\lambda) \Rightarrow (d:\sigma) \preccurlyeq (d+\ell:\lambda) \text{ for all } \sigma \geq 0, \ell \geq 0, \ell \geq 0, d \geq 0. \; (**)$$

P.σ.: It can be seen that the implication also holds in the other direction, and that converxity of ln (D) is equivalento decreasing impatience. You need not show this. □

EXERCISE 5.1.2. Consider intertemporal choice. We only consider dated outcomes $(t:\alpha)$ with $t \geq 0$ and $\alpha \geq 0$ with discounted utility $D(t)U(\alpha)$ as defined in the course. Assume that for all $0 \leq s < \ell$, $0 \leq \sigma < \lambda$, $d > 0$, and $\varepsilon \in \mathbb{R}$ we have:

$$[(s:\sigma) \sim (\ell:\lambda) \text{ and } (s+d:\sigma) \sim (\ell+d+\varepsilon: \lambda)] \implies \varepsilon > 0. \quad (*)$$

Prove that decreasing impatience, as defined in the course, holds.

EXERCISE 5.2.1. We assume generalized hyperbolic discounting $(t{:}x) \mapsto D(t)U(x)$ with $U(x) = \sqrt{x}$ and $D(t) = \frac{1}{(1+at)^{b/a}}$ with $a = 1$ and $b = 0.0352$. We take year as the unit of time. Verify that

$(0{:}100) \sim (1{:}105)$

in the sense that their DU values are so close (DU = 10.00, equality by two digits) that we may take them to be equal.

We now add a delay of $d = 0.5$, and consider

$(0.5{:}100)$ versus $(0.5+1+\varepsilon{:}105)$.

Under stationarity, we would have indifference for $\varepsilon = 0$.

(a) Calculate the present value PV of (0.5:100).

(b) What is the preference between (0.5:100) and (0.5+1:105)? Does this suggest increasing or decreasing impatience?

(c) What is approximately the preference between (0.5:100) and (2:105)?

We redo the calculations bur from now on we assume $a = 2$. Further, $b = 0.0352$ and U remain unchanged. We change the large outcome 105 into 103.943 so as to obtain the starting indifference again. Verify that

$(0{:}100) \sim (1{:}103.943)$

in the sense that their DU values are so close (DU = 10.00, equality by two digits) that we may take them to be equal.

(d) Calculate the present value PV of (0.5:100).

(e) What is the preference between (0.5:100) and (0.5+1:103.943)? Does this suggest increasing or decreasing impatience?

(f) What is approximately the preference between (0.5:100) and (2.5:103.943)?

(g) Compare the results under parts (c) and (f) and, in particular, how much the agent is willing to wait longer to get the larger outcome. Does this suggest that there is more or less *decreasing* impatience when we increase a from 1 to 2?

EXERCISE 5.3.1. Assume quasi-hyperbolic discounting with $\beta < 1$.

(a) Show that stationarity is violated.

(b) Can you show that stationarity is violated with an example that does not have the present, timepoint $t = 0$, involved? No need to elaborate much or write anything, but just give the main argument in one or two spoken sentences.

EXERCISE 5.3.2. We assume quasi-hyperbolic discounting $(t{:}x) \mapsto D(t)U(x)$ with $U(x) = \sqrt{x}$ and

- $t = 0$: $D(t) = 1$ $(= e^{-\delta t})$
- $t > 0$: $D(t) = \beta \times e^{-\delta t}$

with $\delta = 0.01$ and $\beta = 0.95$.

We take year as the unit of time. We have

$(0{:}100) \sim (1{:}113.042)$

in the sense that their DU values are so close (DU = 10.00, equality by two digits) that we take them to be equal. You can, but need not, verify this claim by yourself if you want.

We now add a delay of $d = 0.5$, and consider the indifference

$(0.5{:}100) \sim (0.5{+}1{+}\varepsilon{:}113.042)$.

(a) What would ε be under stationarity? Only give ε, and give no explanation.

(b) Calculate the present value PV of $(0.5{:}100)$.

(c) What is the preference between $(0.5{:}100)$ and $(0.5{+}1{:}113.042)$? Does this suggest increasing or decreasing impatience?

(d) What is, going by two digits, the preference between $(0.5{:}100)$ and $(6.6295{:}113.042)$?

We redo the calculations but from now on we assume $\beta = 0.90$. Here $\delta = 0.01$ and $U(x) = \sqrt{x}$ remain unchanged. We change the large outcome 113.042 into 125.95 so as to obtain the starting indifference again. That is, we now have

$(0{:}100) \sim (1{:}125.95)$

in the sense that their DU values are so close (DU = 10.00, equality by two digits) that we take them to be equal. You can, but need not, verify this claim by yourself if you want. We again add a delay of $d = 0.5$, and consider the indifference

$(0.5{:}100) \sim (0.5{+}1{+}\varepsilon{:}\ 125.95)$.

(e) Calculate the present value PV of $(0.5{:}100)$.

(f) What is the preference between $(0.5{:}100)$ and $(0.5{+}1{:}125.95)$? Does this suggest increasing or decreasing impatience?

(g) What is, going by two digits, the preference between $(0.5{:}100)$ and $(12.036{:}125.95)$?

(h) Compare the results under parts (d) and (g) and, in particular, how much longer the agent is willing to wait for the large reward 125.95 instead of the small reward 100 when the delay $d = 0.5$ is added. Does this comparison suggest that decreasing impatience has become stronger or weaker when we decreased β from 0.95 to 0.90?

EXERCISE 5.4.1. We assume unit invariance discounting $(t{:}x) \mapsto D(t)U(x)$ with $U(x) = \sqrt{x}$ and $D(t) = e^{-(rt)^{\delta}}$ with $r = 0.01$ and $\delta = 0.8$. We take year as the unit of time. Verify that

(0:100) ~ (1:105.152)

in the sense that their DU values are so close (DU = 10.00, equality by two digits) that we may take them to be equal.

We now add a delay of $d = 0.5$, and consider

(0.5:100) versus (0.5+1+ε:105).

Under stationarity, we would have indifference for $\varepsilon = 0$.

(a) Calculate the present value PV of (0.5:100).

(b) What is the preference between (0.5:100) and (0.5+1:105.152)? Does this suggest increasing or decreasing impatience?

(c) What is approximately the preference between (0.5:100) and (1.7635:105.152)?

We redo the calculations but from now on we assume $\delta = 0.5$. $r = 0.01$ and U remain unchanged. We change the large outcome 105.152 into 122.14 so as to obtain the starting indifference again. Verify that

(0:100) ~ (1:122.14)

in the sense that their DU values are so close (DU = 10.00, equality by two digits) that we may take them to be equal.

(d) Calculate the present value PV of (0.5:100).

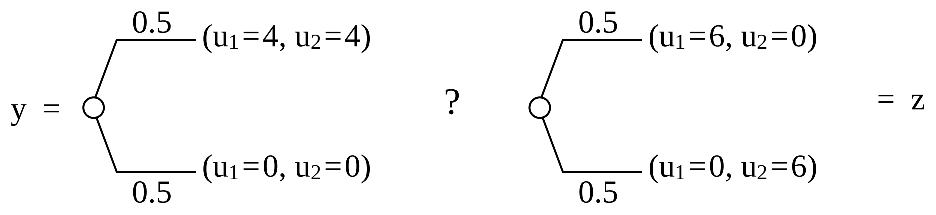
(e) What is the preference between (0.5:100) and (0.5+1:122.14)? Does this suggest increasing or decreasing impatience?

(f) What is approximately the preference between (0.5:100) and (2.9142:122.14)?

(g) Compare the results under parts (c) and (f) and, in particular, how much the agent is willing to wait longer. Does this suggest that the decreasing impatience has become stronger or weaker when we decreased δ from 0.8 to 0.5?

Chapter 6

EXERCISE 6.1.1.

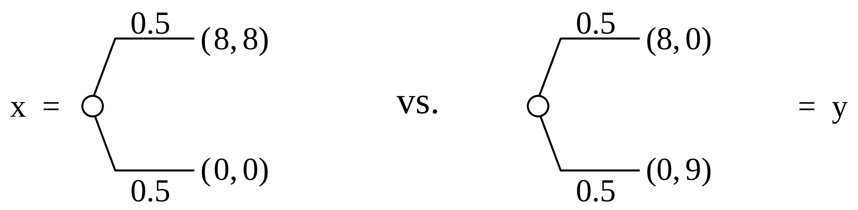


Prospects y and z are probability distributions over social states s. Assume two agents with preferences $\succcurlyeq_1$ and $\succcurlyeq_2$ over prospects. By $\succcurlyeq$ we denote the preference relation over prospects of a benevolent social planner with no self interest. $\succcurlyeq_1$, $\succcurlyeq_2$, and $\succcurlyeq$ maximize expected utility, with utility functions u_1, u_2, and U, respectively. Consider the two prospects in the figure. The left one, y, with probability 0.5 gives a social state with utility 4 for both agents, and with probability 0.5 it gives a social state with utility 0 for both agents. The right prospect, z, is interpreted similarly. The question remark concerns a preference of the social planner discussed below.

(a) Explain that under Pareto optimality the preference of the social planner will be for the right prospect z; i.e., $z \succcurlyeq y$.

(b) Explain how the Fehr-Schmidt (1999) welfare model can accommodate a preference $y \succ z$, assuming $a_1 = a_2 = 0$, and $b_1 = b_2 = b \geq 0$. Thus, (x_1,x_2) is valued by agent 1 as $x_1 - b \times \max\{x_2 - x_1, 0\}$, for instance. So, the agents do not derive negative utility from others being poorer, and they derive the same negative utility from others being richer.

EXERCISE 6.1.2. Abbreviations that you can use: FS for Fehr-Schmidt (welfare model); PO for Pareto optimality; EU for expected utility.



Prospects (x,y, etc.) are probability distributions over social states s. The society consists of two agents, and their preferences over prospects are denoted $\succcurlyeq_1$ and $\succcurlyeq_2$. A benevolent social planner has to decide. Her preference relation over prospects is denoted $\succcurlyeq$. The agents and social planner all maximize EU. By u_1, u_2, and U, we denote their respective utility functions. U is strictly increasing in the sense that strictly improving any outcome in a social state strictly increases U.

In the figure, the left prospect, x, gives a social state with €8 for both agents with probability 0.5, and with probability 0.5 it gives a social state with €0 for both agents. The right prospect, y, is interpreted similarly. The lower branch of y gives (0,9), i.e., €0 for agent 1 and €9 for agent 2.

(a) Assume that the utility of each agent for a social state depends only on the own outcome, with $u_1(\alpha) = u_2(\alpha) = \alpha$ for all α. So, it IS the own outcome. For example, $u_1(0,9) = 0$ and $u_2(0,9) = 9$. What is the preference of the social planner between x and y under PO?

(b) Now assume the FS model for u_2 of agent 2. Agent 2 therefore maximizes EU using u_2 which is FS. Specify when we have $x \succcurlyeq_2 y$. □

Chapter 7

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Chapter 8

EXERCISE 8.1.1. Consider the following game:

	L	R
T	4^1	7^2
M	5^2	5^1
B	7^1	4^2

(a) Assume classical game theory with expected utility. Show that the game has one unique pure Nash equilibrium, which can be obtained by repeated deletion of strictly dominated strategies.

From now on assume that both players maximize neo-additive utility (NAU), with $\sigma =$ 0.75 and $\tau = 0$. That is, they evaluate a strategy by taking 0.75 times its minimal possible utility and 0.25 times its expected utility. Thus, there is strong pessimism, and no optimism. We only consider pure strategies, and no mixed strategies. (For one reason because you were not taught what NAU evaluations of mixed strategies are.)

(b) First show that M strictly dominates T and B in the sense that M is strictly better both if L is played for sure, and if R is played for sure.

(c) Determine the unique pure equilibrium.

EXERCISE 8.1.2. Consider the following 2×2 game. x^y means that player 1, the row player, receives €x and player 2, the column player, receives €y. (Both players have linear utility, $U(\alpha) = \alpha$.) We only consider pure strategies; i.e., randomized strategies cannot be.

	C_1	C_2	C_3
R_1	10^{10}	10^{10}	-100^{-100}
R_2	10^{10}	1^1	0^0
R_3	-100^{-100}	0^0	-100^{-100}

(a) What are the pure equilibria under classical EU? Remember that utility is linear. For this part, only mention the solution, and give no elaboration/justification.

(b) Now assume neo-additive utility (NAU) with pessimism $\sigma = 0.2$ and optimism $\tau = 0.1$, and linear utility. "Possible" means here that the outcome can result from a strategy choice available to your opponent, also if that strategy choice has probability 0. What are the pure equilibria?

(c) We continue to assume the NAU maximization as above, but we now change one entry in the game. The outcome pair for the strategies (R_3,C_3), -100^{-100}, is replaced by x^x, for some x. That is, both players receive x instead of -100 there. See the figure below. For which x does (R_3,C_3) then become an equilibrium? Is (R_2,C_2) then an equilibrium?

(d) In classiccal game theory, expected utility is assumed, i.e., both players maximize expected utility. Then, to find the equilibria of a game, all strictly dominated strategies can be removed. Does this also hold if we assume NAU instead of expected utility? Hint: consider part (b). □

	C_1	C_2	C_3
R_1	10^{10}	10^{10}	-100^{-100}
R_2	10^{10}	1^1	0^0
R_3	-100^{-100}	0^0	x^x

EXERCISE 8.2.1. Consider the following game (a variation of battle of the sexes).

$$
\begin{array}{cc|cc}
 & & q & 1-q \\
 & & L & R \\
\hline
p & T & 0^{0} & 3^{1} \\
1-p & B & 1^{3} & 0^{0}
\end{array}
$$

p denotes the probability of T being played, and q denotes the probability of L being played. Assume Luce's linear choice model, with

$$p = \frac{EU(T)}{EU(T) + EU(B)} \qquad (*)$$

and

$$q = \frac{EU(L)}{EU(L) + EU(R)}\,. \qquad (**)$$

The game is symmetric, and you need only consider symmetric strategies in the sense that $p = q$; so this equality you can assume henceforth. Find the quantal response equilibrium (QRE) with $p = q$.

Chapter 9

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Chapter 10

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Chapter 11

EXERCISE 11.3.1. Assume that a remote island, with 600 people living there, is suddenly hit by a European disease and everyone has caught the disease. Only one of two things can happen for a person having the disease. Either the person has no problems for the next two days, but then suddenly and instantly dies. Or, nothing happens and the person survives. If nothing is done, all 600 people will die. A policy maker has two alternative prospects available and must choose one of the two. Consider two different situations.

SITUATION 1.

Prospect A immediately gives a medicine to everyone, described as follows to the policy maker: exactly 200 people survive.

Prospect B immediately gives a medicine to everyone, described as follows to the policy maker: there is a 1/3 probability that all 600 people will survive, and a 2/3 probability that no-one will survive. So, the policy maker must choose between prospects A and B.

SITUATION 2.

Prospect C immediately gives a medicine to everyone, described as follows to the policy maker: exactly 400 people die.

Prospect D immediately gives a medicine to everyone, described as follows to the policy maker: there is a 1/3 probability that no-one will die, and a 2/3 probability that all 600 people will die. So, the policy maker must choose between prospects C and D.

(a) Which combinations of preferences[7] in the two situations are possible for a rational policy maker?

(b) Which combination of preferences is plausible for a naïve irrational policy maker subject to biases? Just say which without any further comment.

(c) Explain your answer in Part (b).

(d) Does loss aversion play a role in Parts (b) and (c)?

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[7] You can denote combinations of preferences by AC (prospect A is preferred and chosen in Situation 1, and prospect C is preferred and chosen in Situation 2), AD, BC, and BD. We assume that preferences are always strict so that there is never any indifference.

Final Chapter (Conclusion)

EXERCISE 12.1.1. Make up your mind on how what you learned in this course as yet did impact, or can impact, your private life—or at least was supposed to do so according to the teacher. □