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Optimal investment in learning-curve technologies

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ABSTRACT

We study optimal investment in technologies characterized by the learning curve. There are two investment patterns depending on the shape of the learning curve. If the learning process is slow, firms invest relatively late and on a larger scale. If the curve is steep, firms invest earlier and on a smaller scale. We further demonstrate that learning investment differs greatly from investment in technologies without learning effects. Learning investments generate substantial initial losses and are very sensitive to downside risk. We show that the most susceptible to losses and risk are technologies with intermediate speed of learning.

1. Introduction

Technologies in many industries are characterized by learning curves. While producing, firms exploit a process of learning-by-doing that leads to increased efficiency and lower production costs in the future. Investment in these technologies often requires substantial up-front sunk costs and may generate losses in earlier stages. These costs are compensated by the benefits of learning and potential future profits. Whether a new technology turns ultimately profitable depends on the development of its market, which is usually subject to substantial uncertainty. Some recent examples of widely publicized learning investments, i.e. those that are intended to move down the learning curve, include hybrid cars and solar photovoltaic cells.[1]

This paper aims to study a firm's investment in learning-curve technologies under uncertainty. Specifically, we investigate the optimal timing and scale of investment when demand is uncertain and marginal costs decrease with cumulative production. Whereas the literature on investment under uncertainty mainly focuses on the optimal timing o f investment, we also investigate the choice of optimal capacity. This approach is dictated by the fact that scale considerations play a primary role in the presence of the learning curve. A larger capacity allows a higher per-period production rate and a faster reduction of marginal production costs.[2]

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[1] There is ample empirical evidence documenting the presence of learning effects in many other industries. See Wright (1936), Hirsch (1952), Webbink (1977), Zimmerman (1982), Lieberman (1984), Argote et al. (1990), Gruber (1992), Bahk and Gort (1993), Thompson (2001), and Thronton and Thompson (2001) among others.

[2] Capozza and Yuming (1994), Bar-Ilan and Strange (1999), and Dangl (1999) also determine the optimal scale of investment next to the investment timing decision. In contrast to our paper, they do not consider learning effects.

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When the scale of investment is flexible but the timing is not, we find that the presence of the learning curve implies that the firm should invest in a larger capacity. On the other hand, when the timing is flexible but the scale is fixed, the learning curve accelerates the investment. These two observations suggest that investment should occur early and on a large scale to maximize the benefit of learning. However, when timing and scale of investment are simultaneously chosen, a firm faces a trade-off. Investing early, that is, investing at the moment that levels of demand or productivity are still low, implies that only small scale projects are optimal. At the same time, a large scale investment typically requires higher demand or productivity and entails a longer waiting time resulting in foregone profits. The existence of the timing-scale trade-off implies that the firm can optimally exploit the learning curve in two alternative ways that we call "timing option" and "scale option". The timing option entails investing early but with a small productive capacity. By adopting this strategy the firm enters the market early but the small capacity limits the speed of learning. The scale option requires installing a larger production capacity at the cost of delaying the investment. By exploiting the scale option, a firm moves more rapidly along the learning curve when the facility is in place. However, since investment occurs relatively late, it foregoes profits in the short-run.

The resolution of the timing-scale trade-off depends on the steepness of the learning curve. Under slow learning, investment occurs relatively late and on a larger scale, whereas under fast learning it occurs early and on a smaller scale. In the latter case, firms do not need large production rates to substantially reduce marginal costs. Hence, it is optimal to invest soon and install a small capacity. The opposite holds under slow learning, because then optimality implies that a firm should install a larger capacity to reduce marginal costs sufficiently within a given amount of time. Given the larger project size, investment is delayed. It turns out that, when timing is accelerated, scale is inversely U-shaped in the steepness of the learning curve.

To take advantage of learning benefits, firms may undertake learning investments even when current revenue rates are below costs. We show that the optimal investment rule implies that losses at the moment of investment are accepted even for relatively flat learning curves. What matters from an economic point of view is how large accumulated losses are before firms break even. Our analysis indicates that first, the present value of expected initial losses is large. Second, the amount of initial losses is the largest for moderate learning rates. For steep learning curves, the initial level of losses is similar but, because of rapid learning, the break-even point is reached sooner. Third, the losses incurred in early production stages can easily dwarf the initial investment outlays to set up the production facility. Overall, these findings indicate that learning investments can be financially very demanding for firms. This is especially true for technologies with intermediate learning curves.

Learning investment may be particularly exposed to downside risk. For example, new technologies may be superseded by newer technologies and become quickly obsolete. To analyze how downside risk affects optimal investment, we extend the model by introducing the possibility that the project fails and vanishes at a random time. We show that learning investment is very sensitive to this type of risk. Investment is significantly delayed and scale increases with the occurrence of even small levels of downside risk. In contrast, timing and scale of non-learning investment are very insensitive to this type of risk. Furthermore, the value of investment projects with learning curves is decreased more by downside risk. Interestingly, the effects of risk on learning investment are strong for moderate learning curves and steeper curves do not amplify these effects further. The explanation is related to the initial losses associated with learning investment, which are similar for these cases. The threat of the project expiring before any profits materialize distorts the optimal investment and prevents long-term benefits of learning from being fully exploited.

Past theoretical research has recognized the learning curve as a key factor behind firms' production policies and competitive strategies. Some important contributions include Spence (1981), Brueckner and Raymon (1983), Fudenberg and Tirole (1983), Dasgupta and Stiglitz (1988), Majd and Pindyck (1989), Cabral and Riordan (1994, 1997), Dutta and Prasad (1996), Auerswald et al. (2000), and Besanko et al. (2010). However, little attention has been paid to the effects of the learning curve on corporate investment. The closest to our paper is the work by Majd and Pindyck (1989), which studies the optimal production rate under the learning curve and uncertain demand. In their continuous-time model a production facility with fixed capacity is given and no investment decision is analyzed. In contrast, we study flexible investment in a new technology facility to show that the learning curve can significantly affect the choice of investment timing and optimal capacity.

This paper is organized as follows. Section 2 presents the model of investment in the presence of the learning curve, whereas Section 3 analyzes the optimal choice of timing and scale of investment. Section 4 studies initial losses associated with learning investment and Section 5 introduces jump downside risk and investigates its effects on investment. Section 6 is a robustness section which discusses model assumptions and possible extensions. Finally, Section 7 concludes. Proofs are relegated to the Appendix.

2. A model of investment with the learning curve

Time is continuous and labelled by $t \in [0,\infty)$. A firm holds an option to develop a production facility with a technology characterized by a learning curve. The exercise of the investment option involves the decisions of when to invest (timing) and how much capital to install (scale). To focus on learning investment, we assume that at the initial time the firm has no capital invested in the technology. Investment is irreversible and is associated with a lumpy up-front cost. A unit of capital costs i, so investment in K units of capital requires an investment expense of $I(K)=iK$. Once in place, the lifetime of the production facility is assumed to be infinite.

Capital at level K is used to produce output. The production technology is characterized by constant returns to scale and each unit of capital produces one unit of output. The firm produces at its capacity determined by the scale of investment.[3] This implies that per-period output q is always equal to the level of capital, i.e. $q=K$. We discuss the role of this assumption at length in Section 6.2.

Each unit of output is produced at non-negative marginal costs. The learning curve allows the firm to decrease these costs with accumulated experience. At each point in time, marginal costs are constant with respect to the rate of output but, starting from an initial level c, they decline with cumulative output Q. At each time t, Q is given by $\int_0^t q_t\,dt$. To model the learning curve we follow Majd and Pindyck (1989) and set the instantaneous marginal cost equal to

$$c(Q)=ce^{-\gamma Q}, \tag{1}$$

where $\gamma \in [0,\infty)$ is an exogenous parameter that determines the intensity of the learning process. A high (low) γ means that the learning curve is steep (flat).[4]

The firm's output is non-storable and sold at a unit market price denoted by P. The instantaneous profit function is then given by

$$\pi=(P-ce^{-\gamma Q})K. \tag{2}$$

Profits are discounted at rate ρ.

We assume that the price is determined by the inverse demand function[5]

$$P=X-\psi q, \tag{3}$$

where ψ is a strictly positive constant and X is a demand shift parameter that fluctuates according to a geometric Brownian motion with drift μ and variance σ:

$$\frac{dX_t}{X_t}=\mu\,dt+\sigma\,dZ_t. \tag{4}$$

The drift and the discount rate are related such that $\rho>\mu$.[6]

The per-period profit can be written as a function of demand shock X, capital stock K, and cumulative output Q:

$$\pi=(X-\psi K-ce^{-\gamma Q})K. \tag{5}$$

Once the capital is in place, the facility yields an expected discounted stream of profits equal to

$$V(X,K,Q)=\mathbb{E}\left[\int_t^\infty (X_t-\psi K-ce^{-\gamma Q_s})Ke^{-\rho(s-t)}\,ds\,|X_t=X,\ Q_t=Q\right]. \tag{6}$$

Given investment scale K, Q_t is equal to Kt. Using this we obtain that

$$V(X,K,Q)=\frac{XK}{\rho-\mu}-\frac{\psi K^2}{\rho}-\frac{ce^{-\gamma Q}K}{\rho+\gamma K}. \tag{7}$$

Note that the stream of production costs is discounted at the "learning adjusted" rate $\rho+\gamma K$. A larger capacity implies a larger per-period production rate, faster learning and, therefore, a lower discounted stream of costs.

3. Timing and scale of learning investment

3.1. Benchmarks: fixed timing and fixed scale

In the model the firm simultaneously chooses timing and scale of investment. However, we initially consider two benchmark cases. In the first, the firm can only choose the optimal project size without the option to delay the investment. In the second, it has the flexibility to choose the investment timing while the size is fixed.

[3] For simplicity we do not allow for temporary suspension of the production; its role has been studied in Majd and Pindyck (1989). See also Section 6.2 for further discussion.

[4] In the model, we assume that the marginal cost declines monotonically with the cumulative output. Argote et al. (1990), among others, suggest to replace cumulative output with effective knowledge (for a review of the theoretical and empirical literature on learning by doing, see Thompson, 2008). The effective knowledge grows with output but depreciates at some rate with respect to time. Denote by $\widehat{Q}$ the effective knowledge. Argote et al. (1990) suggest that $\widehat{Q}$ follows $d\widehat{Q}=q-\delta\widehat{Q}$, where δ is the rate of depreciation. Estimates of δ reveal wide variations. Darr et al. (1995) argue that the depreciation rate is high in industries with low technological sophistication but it is low in more technologically advanced sectors. In particular, industries in which the learning process is embedded in the capital stock may accumulate knowledge without an economically relevant "forgetting rate", i.e. $\delta\approx 0$. Our specification of the learning curve approximates the learning process in more advanced technological sectors.

[5] The results presented are not driven by the model specification. In particular, all the main results are also present in another popular specification in these types of models: a price taking firm with decreasing returns to scale technology (P_t is an exogenous diffusion process and the rate of production with capital K is K^α, $\alpha<1$). The analysis is available from the authors upon request.

[6] If $\rho-\mu\le 0$, it would be optimal to indefinitely postpone the investment.

Consider the case in which the firm's strategy is limited to the optimal capacity choice. Given the market conditions X, in this scenario the firm chooses the optimal size of investment $\overline{K}$ by solving

$$\max_K [V(X,K,0)-iK].$$

The optimal capacity is implicitly determined by the standard optimality condition that equates the marginal value of an additional unit of capital with the marginal cost:

$$\frac{X}{\rho-\mu}-\frac{2\psi\overline{K}}{\rho}-\frac{c}{\rho+\gamma\overline{K}}+\frac{\gamma c\overline{K}}{(\rho+\gamma\overline{K})^2}=i. \tag{8}$$

The first question that we want to answer is how the speed of the learning process affects the optimal size of the project when investment cannot be delayed. That is, we want to know how γ affects $\overline{K}$. Condition (8) cannot be explicitly solved for $\overline{K}$. However, we can show that the following proposition holds.

Proposition 1. *When the firm can choose the scale but not the timing of investment, the scale $\overline{K}$ is increasing in γ.*

Proposition 1 implies that a more intense learning process increases the scale of investment. Intuitively, a larger γ means a faster reduction of the marginal costs, a larger marginal value of capital and, therefore, a larger optimal capacity.

Consider now the case of a firm that has an option to choose the optimal timing of investment for a project of fixed size K. The firm observes the evolution of the market conditions X and invests at time t^*, where $t^* = \inf\{t : X \geq \overline{X}\}$ and $\overline{X}$ is the demand level that triggers the investment. The investment trigger $\overline{X}$ is optimally chosen by the firm.

Denote by $F(X,K)$ the value of the option to invest. Standard arguments (Dixit and Pindyck, 1994) imply that this option satisfies the ordinary differential equation

$$\tfrac{1}{2}\sigma^2X^2F_{XX}(X,K)+\mu XF_X(X,K)-\rho F(X,K)=0. \tag{9}$$

The general solution of (9) is $A(K)X^{\beta_1}+B(K)X^{\beta_2}$, where β_1 and β_2 are the roots of the fundamental quadratic equation $\frac{1}{2}\sigma^2\beta(\beta-1)+\mu\beta-\rho=0$, i.e.

$$\beta_1=\frac{1}{2}-\frac{\mu}{\sigma^2}+\sqrt{\left(\frac{\mu}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2\rho}{\sigma^2}}>1,$$

$$\beta_2=\frac{1}{2}-\frac{\mu}{\sigma^2}-\sqrt{\left(\frac{\mu}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2\rho}{\sigma^2}}<0.$$

The two coefficients, $A(K)$ and $B(K)$, are to be determined by appropriate boundary conditions. As X reaches zero (its absorbing state), the investment opportunity is foregone forever and the investment option is valueless, i.e. $F(0,K)=0$. This implies that $B(K)=0$. The investment trigger $\overline{X}(K)$ and the coefficient $A(K)$ are obtained from the value matching and smooth pasting conditions

$$F(\overline{X}(K),K)=V(\overline{X}(K),K,0)-iK, \tag{10}$$

$$F_X(\overline{X}(K),K)=V_X(\overline{X}(K),K,0). \tag{11}$$

Substitution of (7) into (10) and (11) eventually yields

$$\overline{X}(K)=\frac{\beta_1(\rho-\mu)}{\beta_1-1}\left(\frac{\psi K}{\rho}+\frac{c}{\rho+\gamma K}+i\right) \tag{12}$$

and

$$F(X,K)=\frac{K}{\beta_1-1}\left(\frac{\psi K}{\rho}+\frac{c}{\rho+\gamma K}+i\right)\left(\frac{X}{\overline{X}(K)}\right)^{\beta_1}. \tag{13}$$

From Eq. (12) the next proposition immediately follows.

Proposition 2. *When the firm can choose the timing but not the scale of the investment, the investment trigger $\overline{X}(K)$ is decreasing in γ.*

When the scale of the project is fixed, a more intense learning process accelerates the investment. This result is also intuitive. For a given capacity, a larger γ implies a higher value of the project so that a lower X is needed to induce the firm to invest.

3.2. Joint determination of timing and scale

Up to this point, we considered the timing and scale dimensions separately. We showed that when the scale of the investment is flexible but the timing is not, a more intense learning process implies that a firm should invest in a larger capacity. Also, we showed that when the timing is flexible but the scale is fixed, the learning curve accelerates the investment. These findings may suggest that investment should occur early and on a large scale. However, timing and scale of investment involve a trade-off. The key observation is that an early investment is feasible only on a small scale. Investing early, i.e. when X is low, implies that the marginal revenue of capital is also low at the time of investment. For this reason, capacity must be small. A large scale is sustainable only if demand is large. Here, we bring the timing and scale dimensions together and see how, in the presence of the learning curve, the timing-scale trade-off is optimally resolved.

Before proceeding with the analysis, we make the following assumption:

Assumption 1. The positive root β_1 of the fundamental quadratic satisfies $\beta_1 > 2$.

As we will show below, Assumption 1 guarantees that a finite solution for the investment problem exists. We elaborate on the economic meaning of this parametric restriction in Section 3.3.[7]

Given the timing rule defined by (12), the firm determines the project scale to maximize the value of its investment option. That is, the optimal capacity $\overline{K}$ maximizes (13). Rearranging the first order condition, $F_K(X,\overline{K})=0$, yields

$$(\beta_1-2)\frac{\psi\overline{K}}{c\rho}-\frac{\rho+\beta_1\gamma\overline{K}}{(\rho+\gamma\overline{K})^2}=\frac{i}{c}, \tag{14}$$

which implicitly defines $\overline{K}$. It is easy to see that a finite positive solution for $\overline{K}$ exists if $\beta_1>2$. Substituting the optimal capacity $\overline{K}$ in (12) gives the value for the optimal investment threshold $\overline{X}(\overline{K})$, also denoted simply by $\overline{X}$.

A closed form expression for the optimal capacity $\overline{K}$, and therefore for the investment trigger $\overline{X}$, is not available. Yet, in the Appendix we show that, when it exists, $\overline{K}$ is uniquely determined by (14). Furthermore, we show the following analytical results regarding the effects of the steepness γ of the learning curve on the scale and timing of investment.

Proposition 3. *The investment trigger $\overline{X}$ is decreasing in γ, $d\overline{X}/d\gamma<0$. The optimal capacity $\overline{K}$ is a non-monotonic function of γ. Specifically, there exists a unique γ^* such that then $d\overline{K}/d\gamma>0$ for $\gamma<\gamma^*$, whereas, then $d\overline{K}/d\gamma<0$ for $\gamma>\gamma^*$.*

Proposition 3 says that investment is accelerated, i.e. the investment trigger is lower, when the learning curve is steeper while the optimal capacity is first increasing and then decreasing with the intensity of the learning process. The effect on the investment trigger is the same as in the model without capacity choice (Proposition 2). However, this result is not obvious because, when timing and capacity are jointly determined, there are two potentially opposing forces at work. The total effect of γ on $\overline{X}$ is given by

$$\frac{d\overline{X}(\overline{K})}{d\gamma}=\frac{\partial\overline{X}(\overline{K})}{\partial\gamma}+\frac{\partial\overline{X}(\overline{K})}{\partial\overline{K}}\frac{d\overline{K}}{d\gamma}. \tag{15}$$

The first term, always negative, is the direct effect of γ on the investment trigger. The second term is the indirect effect of γ through its influence on the optimal capacity $\overline{K}$. The sign of this term is ambiguous and is positive when $\overline{K}$ increases in γ. Potentially, the downward effect of the learning curve on the investment trigger, meaning earlier investment, may be more than compensated by the upward effect due to the larger capacity. Although, Proposition 3 shows that the first effect is always dominant, Eq. (15) highlights a trade-off between timing and scale. A large production scale entails a higher expenditure and tends to delay investment, (partially) counterbalancing the direct effect of γ on the investment trigger. Since, as shown in Section 3.1, the learning curve calls for an early investment on a larger scale, the solution of the timing-scale trade-off is a key aspect concerning investments in learning-by-doing technologies.

In light of the timing-scale trade-off, the investment problem can be presented in the following terms.[8] Consider a firm that has the opportunity to invest in a learning curve technology. The firm has to accomplish a certain level of cumulative production to reach a desired efficiency level, and it can do so in two ways. The first is investing early on a small scale. We call this strategy "exploiting the timing option". The second is setting a larger production rate but investing relatively late. We call this strategy "exploiting the scale option". By exploiting the timing option, the firm starts producing earlier but this comes at the cost of having a small capacity in place and therefore a relatively slower learning. By exploiting the scale option, a firm moves more rapidly along the learning curve when the facility is in place. However, since investment occurs relatively late, it foregoes profits in the short run which is costly due to discounting.

The purpose of our investigation is to answer the following questions. How does the investment strategy in a learning curve technology compare with the one which involves production at constant marginal costs? Does the learning firm exploit the timing or the scale option? To answer these questions, it is first useful to consider the optimal capacity choice in the two limiting cases $\gamma=0$ and $\gamma\to\infty$. In these cases, explicit expressions for $\overline{K}$ can be calculated

$$\overline{K}=\frac{\rho i+c}{(\beta_1-2)\psi}\quad \text{for } \gamma=0 \tag{16}$$

[7] A similar restriction is found in the optimal timing-capacity model of Bar-Ilan and Strange (1999).

[8] We thank an anonymous referee for suggesting this line of interpretation.

and

$$\overline{K} = \frac{\rho i}{(\beta_1 - 2)\psi} \quad \text{for } \gamma \to \infty. \tag{17}$$

It is immediate to see that the optimal capacity for $\gamma \to \infty$ is smaller than for $\gamma = 0$. Since, according to Proposition 3, $\overline{K}$ is initially increasing in γ and then decreasing, this means that there exists a unique $\widehat{\gamma} > 0$ for which the optimal scale investment is the same as the one of a firm that produces at a constant marginal cost ($\gamma = 0$). Proposition 3, in combination with (16) and (17), suggests the following answer to our research questions.

Compared with a firm that produces at constant marginal costs, a learning firm exploits the scale option and installs a larger capacity while (relatively) delaying entry into the market, if the intensity of the learning process is relatively low ($\gamma < \widehat{\gamma}$). In contrast, when the intensity of the learning process is high ($\gamma > \widehat{\gamma}$), a learning firm exploits the timing option by accelerating entry and investing on a smaller scale. We interpret our findings as follows. When the learning curve is almost flat, the learning effect, although present, is weak and the firm needs a high per-period production rate to reduce the marginal cost at a sufficient speed. For this reason, the benefits of a larger capacity outweigh the costs of a delayed investment. On the contrary, when the learning curve is steep, the learning process is fast even with a low production rate. Therefore, the benefits of an additional unit of capital, in terms of an increased speed of learning, are small compared to the cost of delaying the investment. The optimal strategy is to reduce the scale of the project and to enter the market early. Thus, a simple rule of thumb can effectively describe the investment strategy in the presence of the learning curve. If the learning process is slow, invest relatively late and on a larger scale. If the learning process is fast, invest early and on a smaller scale.[9]

To corroborate our interpretation, we provide a numerical example. We set the parameter values as follows. The initial marginal cost is $c = 5$, the discount rate is equal to $\rho = 0.06$, the drift is set equal to $\mu = 0$, the demand parameter is $\psi = 1$, while volatility is $\sigma = 0.1$. Finally, the cost of one unit of capital is $i = 1$.

Fig. 1 presents the effects of γ on scale and timing of investment. The dashed curve in Fig. 1 plots $\overline{K}$ as a function of γ. As confirmed in Proposition 3, the optimal capacity $\overline{K}$ is an inverse-U-shaped function. The steep increase of $\overline{K}$ for low γ has a substantial effect: the scale of investment is larger with learning than without learning up to γ equal to about 0.05. The solid curve shows that $\overline{X}$ is a monotonic and decreasing function of γ. Both timing and scale can be greatly affected by the learning curve. For small learning rates, investment is late and on a large scale. For larger learning rates, say for values of γ above 0.05, investment is taken early and on a relatively small scale.

3.3. Comparative statics

The main focus of this work is on the effects of the learning curve on timing and scale of investment. This section briefly presents the comparative statics effects for the other model parameters and show how the learning effect interacts with two parameters of particular interest, the volatility parameter σ and the demand parameter ψ, to shape the optimal investment strategy of the firm.

Comparative statics results are summarized in (18) and (19). Details of the derivation are shown in Appendix A.5

$$\overline{X}_\sigma > 0, \quad \overline{X}_\mu \lesseqgtr 0, \quad \overline{X}_\rho \lesseqgtr 0, \quad \overline{X}_i > 0, \quad \overline{X}_c > 0, \quad \overline{X}_\psi > 0, \tag{18}$$

$$\overline{K}_\sigma > 0, \quad \overline{K}_\mu > 0, \quad \overline{K}_\rho \lesseqgtr 0, \quad \overline{K}_i > 0, \quad \overline{K}_c > 0, \quad \overline{K}_\psi < 0. \tag{19}$$

Results show that uncertainty delays investment, a standard effect of real options models, but increases the optimal scale. This finding is in line with what is obtained by Bar-Ilan and Strange (1999) and Dangl (1999) in real option models that include capacity choice. The explanation is that, when investment is undertaken at a higher investment trigger, the marginal value of capital at the time of investment is also higher and therefore the optimal capacity level increases. A similar intuition holds for the effects of the investment cost i and the initial marginal production cost c. In general, this mechanism implies that factors that delay investment typically tend to increase the scale. This is not true, however, for the demand parameter ψ. For larger values of ψ investment is delayed but, despite the larger demand, the marginal revenue of production at the time of investment is lower and investment occurs on a lower scale. As Bar-Ilan and Strange (1999), we find that the effect of μ and ρ on both investment timing and scale is ambiguous.

In light of the comparative statics effects and their interpretation, it is useful to reconsider and briefly discuss the economic meaning of Assumption 1. The restriction $\beta_1 > 2$ requires, other things being equal, that the volatility coefficient σ cannot exceed a certain level. The reason is that larger volatility gives rise to a self-reinforcing mechanism such that the optimal investment trigger and scale explode with uncertainty (on this point, see the discussion in Dangl, 1999). As shown above, larger volatility increases the investment trigger. A larger trigger means that, the time of investment, the marginal

[9] In some industries, production requires a minimum scale of investment. To account for this aspect, we could add a constraint to the maximization problem requiring that $\overline{K} \geq \widehat{K}$, where $\widehat{K}$ is the minimum feasible scale of investment. Then, if the optimal capacity in the unconstrained problem is below $\widehat{K}$, we set $\overline{K} = \widehat{K}$ and compute the corresponding investment trigger. Clearly, if the firm cannot reduce its scale below a certain level, it cannot (substantially) anticipate its investment because a sufficiently large demand is needed to make the minimum scale profitable. This would make the timing-scale trade-off less strong.

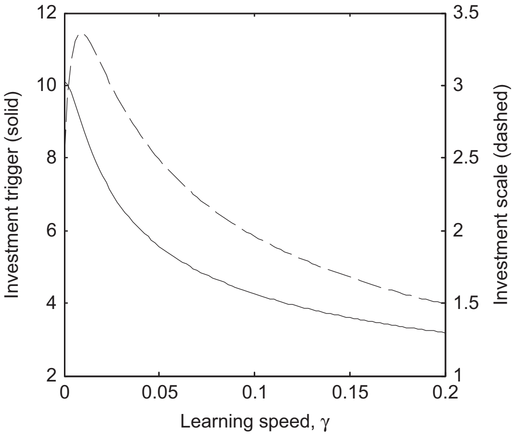

Fig. 1. Optimal scale and investment trigger. Parameter values are $c=5$, $\rho=0.06$, $\mu=0$, $\sigma=0.1$, $\psi=1$, $\sigma=0.1$ and $i=1$.

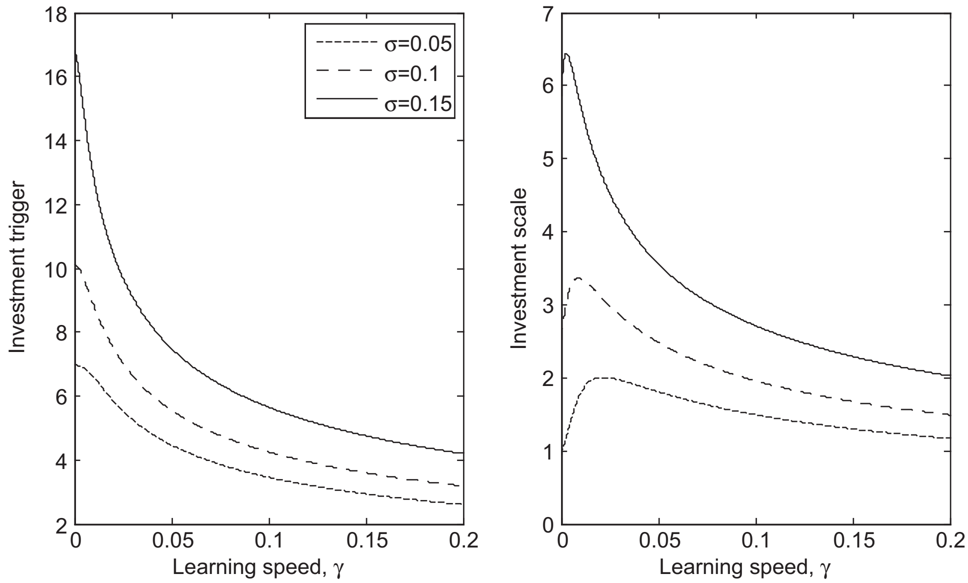

Fig. 2. Optimal scale and investment trigger for different values of volatility σ. Parameter values are $c=5$, $\rho=0.06$, $\mu=0$, $\psi=1$, and $i=1$.

revenue of production is higher so that the optimal scale increases. A larger scale, in its turn, further increases the trigger. This mechanism implies that, for sufficiently large levels of volatility, it is optimal to indefinitely postpone the exercise of the option so that a finite solution for the investment problem does not exist.

To further gain understanding of the mechanisms at work, it is useful to see how some of the comparative statics effects reported in (18) and (19) interact with the learning curve. As illustrative example we study the interaction effects of the learning curve with the volatility coefficient σ and the return to scale parameter ψ. Interaction effects with other parameters turn out to be relatively less instructive. Fig. 2 shows the effect of γ on timing (Panel A) and scale of investment (Panel B) for σ equal to 0.5, 0.1, and 1.5. As expected, for a given γ, both $\overline{K}$ and $\overline{X}$ increase with uncertainty. However, the figure also reveals that the firm is more likely to exploit the scale option when uncertainty is low. This can be seen by observing that the range of values of γ for which the optimal scale for a learning firm is larger than that of the benchmark $\gamma=0$ decreases with σ. The explanation for this result is that, when uncertainty is high, investment occurs already late, i.e. for a high demand, so that the benefits of a larger capacity are more than offset by the costs of further delaying the investment. For this reason, the firm is more likely to exploit the learning curve by choosing the timing option.

Fig. 3 shows the effect of γ on timing (Panel A) and scale of investment (Panel B) for the demand parameter ψ equal to 1, 1.5 and 2. Beside the effect indicated in (18) and (19), the figure also reveals another interesting pattern. When γ is low, changes in ψ greatly affect the scale of investment while the timing is nearly unchanged. This is a further confirmation that

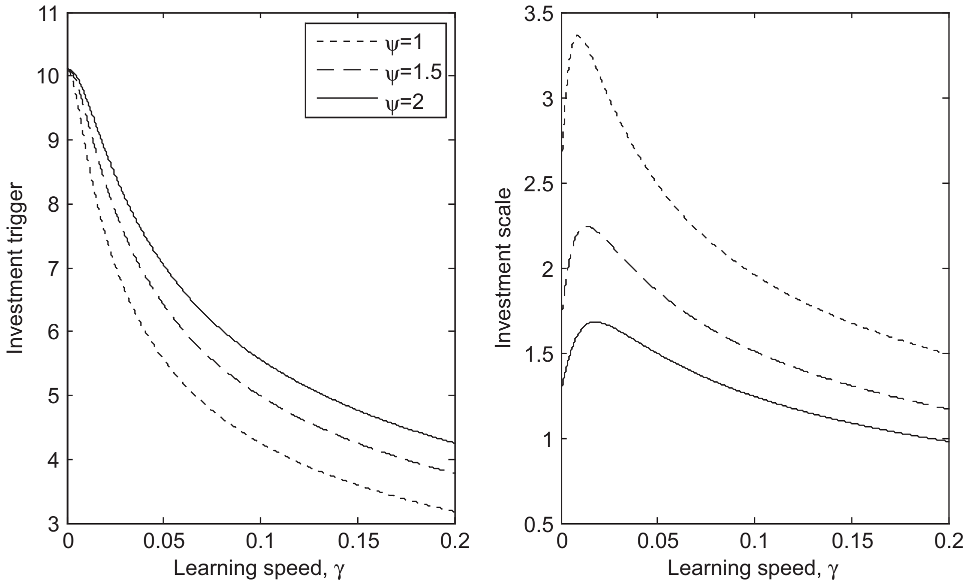

Fig. 3. Optimal scale and investment trigger for different values of ψ. Parameter values are $c=5$, $\rho=0.06$, $\mu=0$, $\sigma=0.1$, and $i=1$.

the scale option is what matters for relatively flat learning curves. In contrast, when γ is large, timing is greatly affected by changes in ψ while the effect on scale is relatively less pronounced. This means that the timing option becomes relatively more important when the learning process is more intense.

4. Initial losses

In the standard real options analysis, firms invest above the break-even point, i.e. they make (substantial) profits from the moment of investment on. However, learning investment brings additional long-term incentives, which may tempt the firm to accept some initial losses. This section, therefore, tries to answer the following questions. Do firms accept some initial losses to benefit from the learning effects? Provided the answer is positive, how large are these losses compared to the initial investment cost? Is steeper learning related to higher initial losses?

Profits or losses at the moment of investment are equal to $(\overline{X}-\psi\overline{K}-c)\overline{K}$. It is easy to verify that initially firms make losses when investing in learning technologies. For our baseline parameter values, this is demonstrated in Fig. 4A. The firm invests at losses already at such low values of γ as 0.015, which implies that the long-run learning incentives are already strong even when the learning curve is relatively flat. Interestingly, after some point the level of losses starts to decrease in γ. This can be explained by the decreasing scale of learning investment in γ (see Fig. 1). Very steep learning implies that the firm invests in small capacity, so losses right after the investment time do not have to be large to support sufficient learning.

Next, we focus on cumulative losses up to the time when the production breaks even. Let us denote by L the expected present value of the stream of losses being incurred before the first time that per-period profits are zero. This will measure how much more capital the firm needs to furnish beyond the initial investment cost.

To simplify notation, at the moment of investment, reset time to $t=0$. Given the profit flow

$$\pi_t = (X_t-\psi\overline{K}-ce^{-\gamma Q_t})\overline{K} = (X_t-\psi\overline{K}-ce^{-\gamma\overline{K}t})\overline{K}$$

the break-even point is achieved at the stopping time $\tau_{BE}=\inf\{t\geq 0: \pi_t=0\}=\inf\{t\geq 0: X_t=\psi\overline{K}+ce^{-\gamma\overline{K}t}\}$. The break-even point can be denoted as a time-dependent threshold on X, namely $X_{BE}(t)=\psi\overline{K}+ce^{-\gamma\overline{K}t}$. Note that π_t can be written as $\pi(X,t)$.

The value of the expected stream of losses from point (X,t) up to the break-even point is denoted by $L(X,t)$. At the moment of the investment it is given by

$$L(\overline{X},0)=E\int_0^{\tau_{BE}}\pi(X_t,t)e^{-\rho t}\,dt.$$

It follows from standard arguments that L must satisfy the partial differential equation

$$\rho L=\mu X L_X+\tfrac{1}{2}\sigma^2X^2L_{XX}+L_t+\pi(X,t),$$

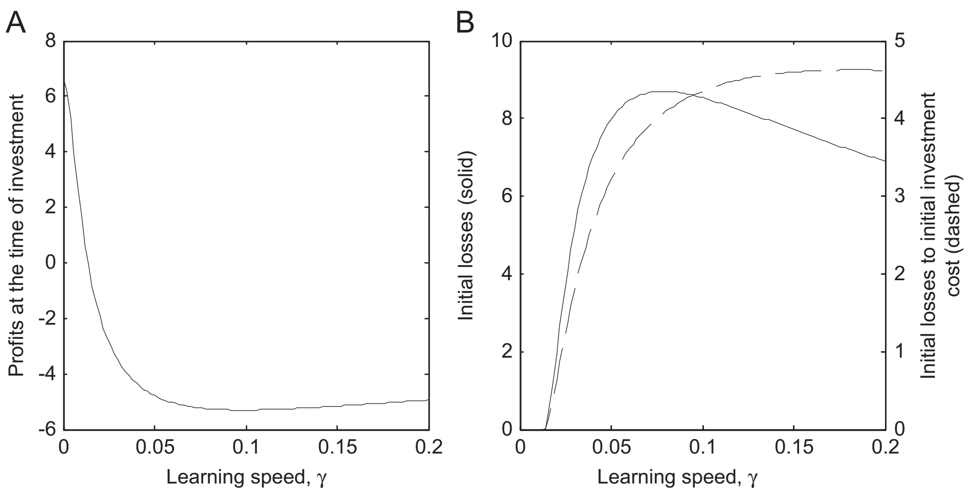

Fig. 4. Profits at the time of investment and the present value of initial losses as a function of learning speed γ. Parameter values are $c=5$, $\rho=0.06$, $\mu=0$, $\psi=1$, $\sigma=0.1$ and $i=1$.

which must be solved subject to a boundary condition at $X_{BE}(t)$, where L should be equal to zero:

$$L(X_{BE}(t),t)=0. \tag{20}$$

This needs to be solved numerically; we apply the finite-difference method.[10]

The results for the baseline parameter values are presented in Fig. 4B. The solid curve plots the present value of initial cumulative losses. Losses are increasing with the intensity of learning for low values of γ and then start decreasing for values of the learning rate γ around 0.08. This pattern originates from the non-monotonicity of instantaneous losses presented in Fig. 4A. However, the non-monotonic shape is much steeper here. This is because with faster learning, the break-even point is reached sooner and the cumulative initial losses decrease. This implies that in terms of required financial slack, investment in technologies with moderate learning effects is most demanding.

It is interesting to compare the initial cumulative losses to the initial cost of investment. The dashed curve in Fig. 4B plots the ratio of these variables for different values of γ. It demonstrates that initial cumulative losses can be very substantial and well exceed the initial cost of investment. In the baseline case the ratio exceeds four and is relatively flat for γ sufficiently large. (The flat shape is caused by the decreased scale of investment for high γ.)

Finally, Fig. 5 shows the effects of uncertainty on the size of the initial cumulative losses. The figure reveals that losses are lower when volatility increases. The explanation is that, when volatility is large, investment occurs late, i.e. for a larger X, and therefore it takes less time to reach the break-even point.

5. Downside risk

The analysis so far assumed that the only source of uncertainty is the diffusion risk in the market demand. However, many investments and technologies may be susceptible to downside jump risk. This may be particularly relevant for learning investment. For example, frontier technologies in such investments can be superseded by even newer technologies.

To examine the effects of downside risk on learning investment, we introduce the possibility that all along the time period after the investment, with probability $\lambda\, dt$ an event can occur that results in the death of the project, where λ is a

[10] Apart from (20), we need other boundary conditions in the (X,t) space. At the absorbing state $X=0$, we have that

$$L(0,t)=-\left(\frac{\varphi\overline{K}}{\rho}+\frac{ce^{-\gamma\overline{K}t}}{\rho+\gamma\overline{K}}\right)\overline{K}.$$

For t high enough, learning is almost completed so profits become independent of cumulative output, and thus L is time-independent. The break-even point is $X_{BE}=\varphi\overline{K}$. $L(X)$. Note that despite marginal production cost is close to zero, for low realization of the demand intercept X (i.e. when $X<\varphi\overline{K}$), profits can be negative. Since profits are independent of cumulative production, L satisfies the ODE, $\rho L=\alpha XL_X+\frac{1}{2}\sigma^2X^2L_{XX}+(X-\varphi\overline{K})\overline{K}$, subject to $L(X_{BE})=0$. This implies that

$$L(X)=\left(\frac{X}{\rho-\alpha}-\frac{\varphi\overline{K}}{\rho}\right)\overline{K}-\left(\frac{X}{\varphi\overline{K}}\right)^{\beta_1}\left(\frac{\varphi\overline{K}}{\rho-\alpha}-\frac{\varphi\overline{K}}{\rho}\right)\overline{K}.$$

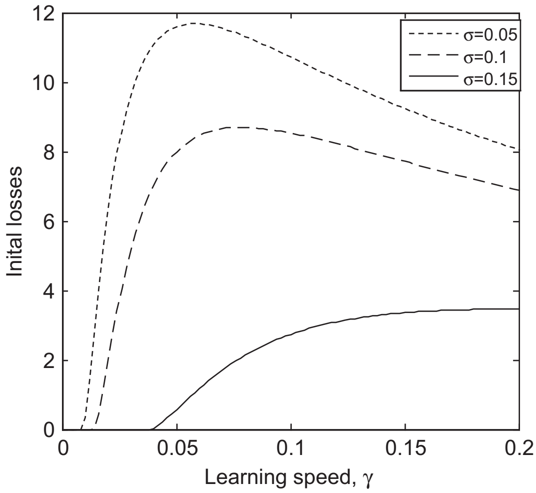

Fig. 5. Present value of initial losses as a function of learning speed γ for different levels of volatility σ. Parameter values are $c=5$, $\rho=0.06$, $\mu=0$, $\psi=1$, and $i=1$.

positive constant. The analysis follows the same steps as those presented in Section 3. Therefore, only the key steps are highlighted here.

The value of the production facility in place is equal to the discounted stream of profits, so that

$$V(X,K,Q)=\frac{XK}{\rho-\mu+\lambda}-\frac{\psi K^2}{\rho+\lambda}-\frac{ce^{-\gamma Q}K}{\rho+\gamma K+\lambda}.$$

Note that the difference with expression (7) is that the discount rate is augmented with the expiry rate λ. For a given capacity K, the investment is optimally undertaken when X reaches the upper trigger $\overline{X}(K)$ given by

$$\overline{X}(K)=\frac{\beta_1(\rho-\mu+\lambda)}{\beta_1-1}\left(\frac{\psi K}{\rho+\lambda}+\frac{c}{\rho+\gamma K+\lambda}+i\right).$$

The optimal scale $\overline{K}$ maximizes the value of the option to invest and is implicitly given by

$$(\beta_1-2)\frac{\psi\overline{K}}{c(\rho+\lambda)}-\frac{\rho+\beta_1\gamma\overline{K}+\lambda}{(\rho+\gamma\overline{K}+\lambda)^2}=\frac{i}{c}.$$

Then the entry trigger equals $\overline{X}=\overline{X}(\overline{K})$.

It is straightforward to derive that both $\overline{X}$ and $\overline{K}$ increase in λ. To verify whether λ can have quantitatively different effects on learning and non-learning investment, we use the baseline parameter values with different learning rates and a range of small realistic values for λ between 0 and 0.1. Fig. 6 presents the results. The solid curve plots the values for investment with no learning effects ($\gamma=0$), the dashed curve represents intermediate learning effects ($\gamma=0.1$), and the dotted curve represents a steep learning curve ($\gamma=0.2$).

Fig. 6A and B shows that investment without learning is rather insensitive to downside risk. The opposite is observed for learning investment. Already for intermediate learning ($\gamma=0.1$), both timing and scale are very sensitive to the presence of downside risk, even when this risk is small. In fact, the effects for the very steep learning curve ($\gamma=0.2$) are not much stronger any more. These observations can be explained by considering the amount of initial losses associated with learning investments. If investment generates losses in early stages, then an early expiry before any long-term gains are realized is particularly costly. Because the losses are the largest for intermediate learning curves, these investments are relatively most sensitive.

Next, we look at the effects on the value of investment in place and on the value of the option to invest. Fig. 6C plots the ratio of the value of investment in place for a range of λ's to the value of investment in place for $\lambda=0$. It shows how much downside risk decreases the value of the production facility at the moment of investment. The figure indicates that the smaller the learning effects, the more the value lost at the moment of investment with the introduction of downside risk. This means that learning investment has more leeway in adjusting timing and scale so that the value at the moment of investment is not so much affected. However, this flexibility comes at a cost. Fig. 6D plots similar ratios but of the value of the option to invest to show how much the investment option value is destroyed by downside risk. Fig. 6D takes the distortion of the terms of investment into account, which is not reflected in Fig. 6C. In this case, the steeper the learning

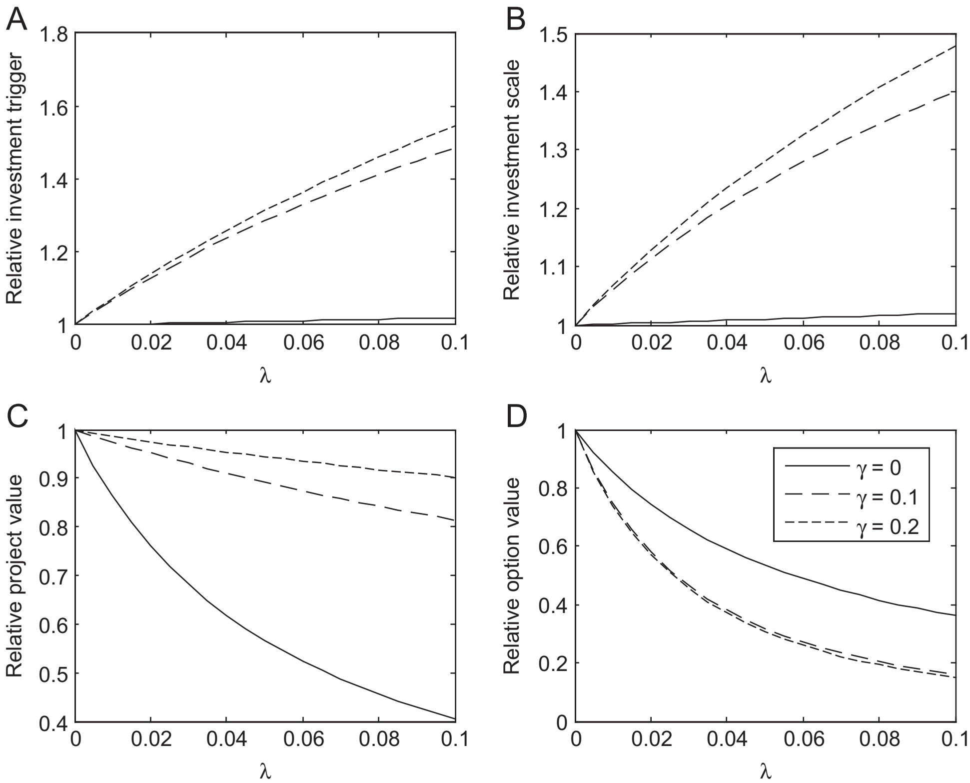

Fig. 6. The effects of downside risk at rate λ. The solid curve plots $\gamma=0$, the dashed curve plots $\gamma=0.1$ and the dotted curve plots $\gamma=0.2$. Other parameter values are $c=5$, $\rho=0.06$, $\mu=0$, $\sigma=0.1$, $\psi=1$, $i=1$, and $X_0=2$.

curve, the more value is lost due to downside risk. This is because learning investments are substantially delayed, which helps to maintain the value of the project at the investment time (Fig. 6C), but weakens the learning potential and destroys the value of the option to invest (Fig. 6D). It is worth noting that the difference between the two learning cases with $\gamma=0.1$ and $\gamma=0.2$ are very small, which shows again that investments in technologies with moderate learning effects are relatively most vulnerable.

6. Robustness

6.1. Expansion option

In the analysis we assumed that the production scale of the firm is determined once and for all by the initial investment decision. This is a simplification as, in reality, firms have often the opportunity to increase their capacity should market demand rise in the future. Another class of models, for example Pindyck (1988), is concerned with the capacity expansion of the firm. We believe that our approach best suits the study of investment decision in learning-curve technologies for at least two reasons.

First, it is well known that, at the micro level, equipment investment has a lumpy nature as firms often increase their capacity by not just expanding existing production facilities but rather by building new plants. Furthermore, the most recent empirical literature on the learning curve focused almost exclusively on the effects of learning-by-doing at the plant level (see Thompson, 2011 for a comprehensive survey), while there is no compelling evidence on the existence and economic importance of inter-plant learning spillovers. Thus, although for simplicity we always refer to a "firm", our study can be considered to be a plant-level analysis. Second, learning-by-doing has a finite time dimension beyond which the benefits in terms of efficiency become negligible (Bahk and Gort, 1993). Within a limited time horizon, even assuming that firms increase capacity by expanding existing production facilities, the scale of a plant is likely to remain stable. For these reasons, a capacity expansion model seems to be not appropriate for the economic problem under consideration.

However, to further check the robustness of our results, we also study a model in which the firm can increase its capacity. Here, we do not present a full description of the analysis and just sketch the main findings. In this extension, the capacity of the firm is determined in two stages. In the first stage, the firm chooses timing and scale of investment as in the model of Section 3. In the second stage, the firm has the opportunity to increase its capacity but without flexibly choosing the timing of investment. Specifically, we assume that the firm has a now-or-never opportunity to increase its scale when the learning process is completed. To do so, we assume that the learning process is not infinite but stops when marginal cost reaches a lower bound $\underline{c} < c$. Findings of this two-stage model substantially confirm the results of Proposition 3. In the first stage, the investment trigger is monotonically decreasing in γ while the optimal scale is first increasing and then decreasing in γ. The interpretation is also the same. When the learning curve is almost flat, the learning effect is weak and the firm needs a high per-period production rate to reduce the marginal cost at sufficient speed. The opposite holds for steeper learning curves.

6.2. Full capacity assumption

In the model we assume that the firm always produces at full capacity, i.e. $q=K$. This, coupled with a linear demand function, implies that the price can turn negative for low realizations of the demand intercept X. Clearly, producing at full capacity in every state of the world is not optimal. If a firm can costlessly suspend its production, it chooses, at any time, an optimal production rate q^* which maximizes instantaneous profits. This means that, when demand is low, some capacity units remain idle, i.e. $q^* < K$. In practice, a fully flexible production rate is hardly realistic. In many industries firms make production plans before the actual realization of market demand. Goyal and Netessine (2007) point out that firms may find it difficult to produce below capacity due to fixed costs associated with flexibility and commitments to suppliers (see also articles cited therein). Even when firms can keep some capacity idle, a temporary suspension of production is often costly. This is the case, for example, because of the maintenance costs needed to avoid deterioration of the equipment.

When a firm faces a learning curve, the full capacity assumption is perhaps closer to reality due to the fact that, conditions under which it is optimal to produce at full capacity are less stringent than in the constant marginal costs case. In fact, while a non-learning firm maximizes current profits by equating marginal revenue to marginal cost, a firm that faces the learning curve chooses a production rate such that marginal revenue equals marginal cost minus the benefits of future cost savings from current production. Thus, its production rate will be typically higher.

One may argue that the full capacity assumption has a key role in determining the large up-front losses documented in Section 4. In principle, with a flexible production rate, a firm may be able to reduce the amount of initial losses by producing below capacity. However, since a firm produces when marginal revenue is below the marginal cost, the presence of substantial up-front losses is an inherent feature of investments in learning-curve technologies. This feature will also be present when the firm is able to produce below capacity. Here, it is important to note that, compared to full capacity production, producing below capacity reduces the speed of learning, where the effect of the latter is that it increases the amount of losses.

7. Conclusions

This paper investigates optimal investment behavior in learning curve technologies. Marginal cost decreases with cumulative production and the firm under consideration operates in an uncertain output market with fluctuating demand. The firm can invest in learning curve technologies by exploiting two alternative strategies. It can either invest early and on a small scale or invest late but enter the market with a larger productive capacity. We show that, if the learning curve is flat, the latter strategy is optimal. On the other hand, when the learning curve is steep, early and small scale investments are preferred. This result holds even when the firm can further expand its capacity in later stages of production.

We also show that learning investment is associated with large expected losses in the early stages after the firm undertook the investment. The reason is that, due to learning, production costs reduce over time, implying that in the beginning they are still high. The implication of the occurrence of initial losses is that learning investment is very sensitive to downside risk in the sense that an event can occur leading to the end of the project at a time too early for learning to have caused sufficient cost reductions. The expected losses and distortions are particularly strong for investment with moderate learning curves.

Despite its focus on optimal firm decisions, our analysis has some clear policy implications. Investing in technologies with very steep learning curves, like many information technologies, can be efficiently undertaken by firms. However, investing in technologies with moderate learning curves is more difficult and more financially demanding for firms. Frictions, for example financing constraints, may easily lead to suboptimal investments in these technologies. If there are some positive externalities of technological investments, such as learning spillovers or positive environmental effects, then suboptimal investment in such technologies may be especially costly from a welfare perspective. This category of technologies, i.e. with moderate learning and large externalities, may include some energy technologies. In this case, public support of investment, e.g., in the form of guaranties, may be warranted.

Acknowledgement

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Appendix A. Proofs

A.1. Proof of Proposition 1

Totally differentiating condition (8) and rearranging yields $d\overline{K}/d\gamma = -V_{K_\gamma}(X,\overline{K},0)/V_{KK}(X,\overline{K},0)$. From (7), $V_{K\gamma}(X,\overline{K},0) = 2\rho c\overline{K}/(\rho+\gamma\overline{K})^3 > 0$. Also, optimality rules out the possibility that the firm will invest in the region in which marginal returns to capital are non-decreasing. i.e. $V_{KK}(X,K,0) \geq 0$. If this would be the case, the firm would be better off by increasing its scale. It follows that optimality must necessarily imply that $V_{KK}(X,K,0) < 0$. Hence, the effect of γ on the optimal capacity, $d\overline{K}/d\gamma$, is positive.

A.2. Proof of Proposition 2

In the text.

A.3. Optimal capacity: existence and uniqueness

Differentiating (13) with respect to K yields the first order condition

$$F_K(X,\overline{K}) = \frac{1}{\beta_1-1}\left(\frac{X}{\overline{X}(K)}\right)^{\beta_1}\left\{\frac{\psi\overline{K}}{\rho}+\frac{c}{\gamma\overline{K}+\rho}+i-(\beta_1-1)\left[\frac{\psi\overline{K}}{\rho}-\frac{\gamma\overline{K}c}{(\rho+\gamma\overline{K})^2}\right]\right\} = 0. \tag{21}$$

Rearranging the term between the curly brackets yields condition (14). Existence of a positive finite solution to (14) was argued in the text and requires that $\beta_1 > 2$.

The second order condition for the maximum at the solution $\overline{K}$ is

$$F_{KK}(X,\overline{K}) = \frac{1}{\beta_1-1}\left(\frac{X}{\overline{X}(K)}\right)^{\beta_1}\left\{(2-\beta_1)\left[\frac{\psi}{\rho}-\frac{\gamma c}{(\gamma\overline{K}+\rho)^2}\right]+(1-\beta_1)\frac{\gamma^2\overline{K}c}{(\gamma\overline{K}+\rho)^3}\right\} < 0. \tag{22}$$

To verify that the inequality holds, we use the first order condition. Note that, because the first three terms between the curly brackets of (21) are positive and $\beta_1 > 2$, the first order condition requires that

$$\frac{\psi\overline{K}}{\rho}-\frac{\gamma\overline{K}c}{(\gamma\overline{K}+\rho)^2} > 0. \tag{23}$$

Employing (23) and $\beta_1 > 2$ we confirm the inequality in (22). Given that $F(X,K)$ is a continuous and smooth function, local concavity at the stationary points implies that there is a unique stationary point. Hence, $\overline{K}$ (when it exists) is unique and maximizes $F(X,K)$.

A.4. Proof of Proposition 3

The effect of γ on the optimal capacity is given by $d\overline{K}/d\gamma = -F_{K\gamma}(X,\overline{K})/F_{KK}(X,\overline{K})$. Differentiating and rearranging the first order condition (21) with respect to γ yields

$$F_{K\gamma}(X,\overline{K}) = \frac{1}{\beta_1-1}\left(\frac{X}{\overline{X}(K)}\right)^{\beta_1}\frac{c\overline{K}}{(\rho+\gamma\overline{K})^3}[\rho(\beta_1-2)-\beta_1\gamma\overline{K}]. \tag{24}$$

Recalling that $F_{KK}(X,\overline{K}) < 0$, the optimal capacity increases in γ when $F_{K\gamma}(X,\overline{K}) > 0$ and decreases otherwise. The sign of $F_{K\gamma}(X,\overline{K})$ is identical to the sign of the term between the square brackets. Define $\Gamma(\gamma) = \rho(\beta_1-2)-\beta_1\gamma\overline{K}$. The optimal capacity $\overline{K}$ is an inverse-U-shaped function of γ as claimed in the proposition if $\Gamma(\gamma) > 0$ for low values of γ, $\Gamma(\gamma) < 0$ for high values of γ and if there is a unique value of γ which satisfies $\Gamma(\gamma) = 0$.

Assume that (14) holds and that $\beta_1 > 2$ so that a finite positive solution for $\overline{K}$ exists. When γ is small, given that $\lim_{\gamma\to 0}\overline{K}$ is bounded, $\Gamma(\gamma)$ is positive. Assume, now, that $\overline{K}$ is always increasing in γ. This assumption requires that $\Gamma(\gamma) > 0$ for every combination of γ and $\overline{K}$. But given that $\rho(\beta_1-2)$ is constant, for sufficiently large values of γ, it must be that $\Gamma(\gamma) < 0$, contradicting the initial hypothesis. Hence, for large values of γ there exists a region where $d\overline{K}/d\gamma < 0$, i.e. $\Gamma(\gamma) < 0$.

Finally, we show that γ^* is unique. Differentiating $\Gamma(\gamma)$ with respect to γ yields

$$\Gamma'(\gamma) = -\beta_1\overline{K}-\beta_1\gamma\, d\overline{K}/d\gamma,$$

which is negative if $d\overline{K}/d\gamma \geq 0$, that is if $\Gamma(\gamma) \geq 0$. It follows that there exists only one value of γ that satisfies $\Gamma(\gamma) = 0$.

The total effect of γ on the optimal investment trigger $\overline{X}$ is given by

$$\frac{d\overline{X}}{d\gamma} = \frac{\partial\overline{X}(\overline{K})}{\partial\gamma}+\frac{\partial\overline{X}(\overline{K})}{\partial\overline{K}}\frac{d\overline{K}}{d\gamma} = \frac{\partial\overline{X}(\overline{K})}{\partial\gamma}-\frac{\partial\overline{X}(\overline{K})}{\partial\overline{K}}\frac{F_{K_\gamma}(X,\overline{K})}{F_{KK}(X,\overline{K})}. \tag{25}$$

Rearranging, we obtain

$$\frac{d\overline{X}}{d\gamma} = -\frac{2(\rho-\mu)(\beta_1-1)c\psi\gamma\overline{K}^2}{(\beta_1-2)[\psi(\gamma\overline{K}+\rho)^3-c\gamma\rho^2]+\beta_1 c\gamma^2\rho\overline{K}}. \tag{26}$$

Since the first order condition (21) requires $\psi/\rho-\gamma c/(\rho+\gamma\overline{K})^2>0$, it follows that the term between the square brackets in the denominator is always positive. Since $\beta_1>2$, it follows that $d\overline{X}/d\gamma<0$.

A.5. Comparative statics

Here, we derive the comparative statics results in (18) and (19). Consider the effect on $\overline{K}$. The effect of a parameter x on the optimal capacity is given by $d\overline{K}/dx = -F_{Kx}(X,\overline{K})/F_{KK}(X,\overline{K})$. Also, recall that $\partial\beta/\partial\sigma<0$, $\partial\beta/\partial\mu<0$, and $\partial\beta/\partial\rho>0$. Differentiating condition (21) yields

$$F_{K\sigma} = -\frac{1}{\beta_1-1}\left(\frac{X}{\overline{X}(\overline{K})}\right)^{\beta_1}\frac{\partial\beta}{\partial\sigma}\left[\frac{\psi\overline{K}}{\rho}-\frac{\gamma\overline{K}c}{(\gamma\overline{K}+\rho)^2}\right]>0,$$

$$F_{K\mu} = -\frac{1}{\beta_1-1}\left(\frac{X}{\overline{X}(\overline{K})}\right)^{\beta_1}\frac{\partial\beta}{\partial\mu}\left[\frac{\psi\overline{K}}{\rho}-\frac{\gamma\overline{K}c}{(\gamma\overline{K}+\rho)^2}\right]>0,$$

$$F_{K\rho} = \frac{1}{\beta_1-1}\left(\frac{X}{\overline{X}(\overline{K})}\right)^{\beta_1}\left\{(\beta_1-2)\frac{\psi\overline{K}}{\rho^2}-\frac{c}{(\gamma\overline{K}+\rho)^2}-2\frac{(\beta_1-1)c\overline{K}}{(\gamma\overline{K}+\rho)^3}-\frac{\partial\beta}{\partial\rho}\left[\frac{\psi\overline{K}}{\rho}-\frac{\gamma\overline{K}c}{(\gamma\overline{K}+\rho)^2}\right]\right\}\lesseqgtr 0,$$

$$F_{Ki} = \frac{1}{\beta_1-1}\left(\frac{X}{\overline{X}(\overline{K})}\right)^{\beta_1}>0,$$

$$F_{Kc} = \frac{1}{\beta_1-1}\left(\frac{X}{\overline{X}(\overline{K})}\right)^{\beta_1}\left[\frac{1}{\gamma\overline{K}+\rho}+(\beta_1-1)\frac{\gamma\overline{K}}{(\gamma\overline{K}+\rho)^2}\right]>0,$$

$$F_{K\psi} = -\frac{1}{\beta_1-1}\left(\frac{X}{\overline{X}(\overline{K})}\right)^{\beta_1}(\beta_1-2)\frac{\psi}{\rho}<0.$$

Using the above cross-derivatives and the fact that $F_{KK}(X,\overline{K})<0$, the effects on the optimal capacity listed in (18) and (19) follow.

Consider now the effect on $\overline{X}$. Define $\Theta=\beta_1/(\beta_1-1)$ and note that $\Theta_\sigma>0$, $\Theta_\mu>0$ and $\Theta_\rho<0$. For K constant, it holds that

$$\overline{X}_\sigma(K) = (\rho-\mu)\Theta_\sigma\left(\frac{\psi K}{\rho}+\frac{c}{\gamma K+\rho}+i\right)>0,$$

$$\overline{X}_\mu(K) = \left(\frac{\psi K}{\rho}+\frac{c}{\gamma K+\rho}+i\right)[\Theta_\mu(\rho-\mu)-\Theta]\lesseqgtr 0,$$

$$\overline{X}_\rho(K) = \left(\frac{\psi K}{\rho}+\frac{c}{\gamma K+\rho}+i\right)[\Theta_\rho(\rho-\mu)+\Theta]-\Theta(\rho-\mu)\left[\frac{\psi K}{\rho^2}+\frac{c}{(\gamma K+\rho)^2}\right]\lesseqgtr 0,$$

$$\overline{X}_i(K) = \Theta(\rho-\mu)>0,$$

$$\overline{X}_c(K) = \Theta\frac{(\rho-\mu)}{\gamma K+\rho}>0,$$

$$\overline{X}_\psi(K) = \Theta\frac{(\rho-\mu)K}{\rho}>0.$$

We have to evaluate the sign of the derivatives at $\overline{K}$. As an illustrative example, consider the effect of volatility. Since $\overline{K}_\sigma>0$, $\overline{X}_\sigma(K)>0$ and $\overline{X}_K(K)>0$, the sign of $\overline{X}_\sigma$ is unambiguously positive. The same reasoning holds for $\overline{X}_i$ and $\overline{X}_c$. On the contrary, we cannot analytically pin down the sign of $\overline{X}_\mu$, $\overline{X}_\rho$ and $\overline{X}_\psi$, and these effects are found numerically.

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