A Context-Dependent Model of the Gambling Effect

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This paper presents a context-dependent theory of decision under risk. The relevant contextual factor is the presence of a riskless lottery in a preference comparison. The theory only deviates from expected utility if the set of options contains both riskless and risky lotteries. The main motivation for the theory is to explain the gambling effect. Contrary to previous theories of the gambling effect, the present theory is consistent with stochastic dominance. It can, however, violate transitivity. The theory allows for a decomposition of the interaction between risk aversion and gambling aversion and thereby extends the classical Arrow-Pratt measure of risk aversion.

(*Nonexpected Utility; Gambling Effect; Risk Aversion; Intransitivity*)

1. Introduction

It is by now widely recognized that expected utility (EU) is not a good descriptive theory of decision under risk. Violations of EU are especially likely when a preference comparison is made between a risky and a riskless lottery (Kahneman and Tversky 1979, Cohen and Jaffray 1988). The common consequence effect and the common ratio effect (Allais 1953) are well-known examples of such EU violations. Violations are much less pronounced when both lotteries are risky (Camerer 1992). This preference pattern can be explained by the existence of a "gambling effect," i.e., the effect that people's processing of lotteries changes if one of the lotteries is riskless.

The possibility of a gambling effect was already mentioned by von Neumann and Morgenstern (1944, p. 28). They felt, however, that an axiomatization of the gambling effect was impossible (see also Tversky 1967). Fishburn (1980), Schmidt (1998), and Diecidue et al. (2001) proposed axiomatic models of the gambling effect in decision under risk. Luce and Marley (2000) developed a model of the gambling effect in decision under uncertainty. Diecidue et al. (2001) showed that any model of the gambling effect has to violate an elementary rationality condition. In their model, as in the models of Fishburn (1980), Schmidt (1998), and Luce and Marley (2000), this condition is stochastic dominance. Diecidue et al. mention the possibility that transitivity can be abandoned instead of stochastic dominance, but only propose an ad hoc manner of doing so, namely through an editing operation.

Stochastic dominance is generally considered normative, and therefore the main application of the above models is descriptive. It is legitimate, however, to ask to what extent the violations of stochastic dominance predicted by these models are really descriptive. Empirical evidence has shown that people violate stochastic dominance in decision situations where stochastic dominance is not transparent, but behave according to stochastic dominance when stochastic dominance is clear. Based on a review of the literature, Starmer (2000) notes as a stylized fact that "very few people will choose a stochastically dominated option from a choice set when it is transparently obvious that the option is dominated" (p. 360). The aforementioned

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models of the gambling effect imply violations of stochastic dominance only when a riskless lottery is compared with a risky lottery which has no outcomes worse than the outcome of the riskless lottery. That is, they imply violations of stochastic dominance only when stochastic dominance is transparent. Therefore, the descriptive applicability of these models seems limited.

The present paper proposes an alternative theory of the gambling effect that is consistent with stochastic dominance. The theory assumes that preferences are context dependent, i.e., preferences depend on the other alternatives in the choice set. We distinguish two different contexts, one in which at least one lottery in a preference comparison is riskless and one in which both lotteries in a preference comparison are risky. We assume that people use different evaluation processes when they compare a riskless lottery with a risky lottery than when they compare two risky lotteries. In both cases the evaluation is performed by expected utility, but the utility function may differ between the two cases. The presence of a riskless lottery makes people more aware of the risk in the other lottery and changes their attitude towards risk. This prediction is in line with the empirical finding that measurements comparing risky with riskless lotteries yield systematically different (more concave) utility functions than measurements that invoke only risky lotteries (McCord and de Neufville 1986, Wakker and Deneffe 1996). Context dependence of preferences distinguishes our model of the gambling effect from the models of Fishburn (1980), Schmidt (1998), Luce and Marley (2000), and Diecidue et al. (2001), in which preferences are independent of the context. Several studies show empirical support for context-dependent preferences (Huber et al. 1982, Huber and Puto 1983, Wedell 1991, Simonson and Tversky 1992, Tversky and Simonson 1993). Marley (1991), Lakshmi-Ratan et al. (1991), and Tversky and Simonson (1993) proposed models of context-dependent preferences in different settings than considered in this paper.

As noted, our model does not violate stochastic dominance (see Corollary 4.2). A consequence of ruling out these violations of stochastic dominance, however, is that the model allows violations of transitivity (see Theorem 4.3). The possibility of intransitivities limits the prescriptive applicability of our model. Even though some authors have argued that intransitive preferences can be normative (Loomes and Sugden 1982, Fishburn 1982, Sugden 1985, Anand 1987, Fishburn 1991), intransitivities are commonly regarded as irrational (Luce 2000). The latter view is confirmed by experimental evidence that people want to correct intransitivities when these are pointed out to them (MacCrimmon 1968). The main application of our model is therefore descriptive. Our model allows violations of transitivity in comparisons involving both riskless and risky lotteries. Loomes et al. (1991) find that such violations of transitivity frequently occur. The most common violation of transitivity in their study is a violation allowed by our model. This evidence supports the descriptive applicability of our model.

Several models of intransitive preferences exist. Luce (1956) attributes failures of transitivity to limited discriminatory ability. Regret theory (Bell 1982, Loomes and Sugden 1982, Sugden 1993) and Fishburn's (1982) skew-symmetric bilinear (SSB) theory retain precise discriminability but allow preferences over lotteries to be cyclic ($P_1 \succ P_2 \succ \cdots \succ P_n \succ P_1$). Like regret theory and SSB theory, our model is a precise discriminability theory. An advantage of our model over regret theory and SSB theory is that it is entirely consistent with Allais' common consequence effect. Regret theory and SSB theory cannot explain the common consequence effect when alternatives are statistically correlated. Empirical studies show evidence of the common consequence effect both when alternatives are independent and when they are correlated (Starmer 1992, Groes et al. 1999).

In what follows, §2 introduces notation and describes the model of this paper, referred to as the gambling effect model. Section 3 gives a preference characterization of the gambling effect model. In §4 we show that the gambling effect model is consistent with stochastic dominance, but violates transitivity. An important advantage of the gambling effect model for practical decision analysis is that its elicitation is straightforward. Section 5 describes the elicitation procedures. Section 6 characterizes the empirically observed case that the utility function that is applied in preference comparisons where at least one lottery is riskless is more concave than the utility function that is applied in comparisons where all lotteries are risky. In EU, risk attitude properties are useful in characterizing the class of applicable utility functions, in reducing the assessment effort, and in analyzing behavior under risk. The Arrow-Pratt measure of risk aversion in particular has proved to be an important tool for characterizing and comparing individual behavior. Section 7 shows how the Arrow-Pratt measure of risk aversion and other important concepts such as aversion to mean-preserving spreads and the risk premium can be extended to the gambling effect model. The gambling effect model allows for a separation between "pure" risk attitudes that are observed in comparisons between risky lotteries and gambling attitudes that are observed in comparisons between riskless and risky lotteries. This separation makes it possible to identify in theoretical applications which results are due to pure risk attitude and which are due to gambling attitude. Properties of pure risk attitude and gambling attitude can be exploited to further specialize the gambling effect model and to analyze individual behavior much in the same way as they are used in expected utility. Section 8 concludes. Proofs are relegated to the appendix.

2. Notation and Outline of the Model

Let $\mathscr{X}$ be a set of outcomes. $\mathscr{X}$ can denote any type of outcomes. We restrict attention to decision under risk, hence probabilities are given. A *lottery* $(p_1, x_1; \dots; p_n, x_n)$ yields outcome $x_i \in \mathscr{X}$ with probability $p_i, i = 1, \dots, n$. Binary lotteries are denoted $(x, p; y)$. By $\mathscr{P}$ we denote the set of all *simple lotteries* over $\mathscr{X}$, i.e., lotteries that assign positive probabilities to a finite number of outcomes. A consequence $x \in \mathscr{X}$ is identified with the *riskless lottery* $(1, x)$. Therefore, $\mathscr{X}$ is identified with the subset of $\mathscr{P}$ containing all riskless lotteries. The remaining set of *risky lotteries* $\mathscr{R} = \mathscr{P} - \mathscr{X}$ contains those lotteries that assign positive probabilities to at least two different outcomes.

A preference relation $\succcurlyeq$ is defined over $\mathscr{P}$. As usual, $\succ$ denotes the asymmetric part of $\succcurlyeq$ (strict preference) and $\sim$ the symmetric part (indifference). Throughout, we assume that $\mathscr{X}$ contains at least three mutually nonindifferent outcomes. We further assume that each lottery $P \in \mathscr{P}$ has a *certainty equivalent*, i.e., for all $P \in \mathscr{P}$ there exists an outcome $x_p \in \mathscr{X}$ such that $x_p \sim P$. A real-valued function V on $\mathscr{P}$ *represents* $\succcurlyeq$ if for all $P, Q \in \mathscr{P}$, $V(P) \geq V(Q)$ if and only if $P \succcurlyeq Q$. The *gambling effect model* holds if there exist real-valued utility functions $v(\cdot)$ and $u(\cdot)$ defined on $\mathscr{X}$, such that for all $P, Q \in \mathscr{P}$ with $\{x_1, \dots, x_n\}$ the joint support of P and Q,

(i) if P and $Q \in \mathscr{R}$, then $P \succcurlyeq Q$ iff $\sum_{i=1}^{n} p_i \cdot u(x_i) \geq \sum_{i=1}^{n} q_i \cdot u(x_i)$;

(ii) if P or $Q \in \mathscr{X}$, then $P \succcurlyeq Q$ iff $\sum_{i=1}^{n} p_i \cdot v(x_i) \geq \sum_{i=1}^{n} q_i \cdot v(x_i)$.

Thus, the individual's preferences depend on the decision context. If both lotteries are risky, then the decision maker behaves as an expected utility maximizer with von Neumann-Morgenstern utility function $u(\cdot)$. If one of the lotteries is riskless, then the decision maker perceives the risk differently and behaves as an expected utility maximizer with von Neumann–Morgenstern utility function $v(\cdot)$. The utility function in EU theory is unique up to positive linear transformations, i.e., for all $x \in \mathscr{X}$, two utility functions $u(\cdot)$ and $u^*(\cdot)$ represent the same preference ordering if and only if $u(x) = a \cdot u^*(x) + b$ with $a > 0$ and b real. Consequently, violations of expected utility can occur in the gambling effect model in case $u(\cdot)$ and $v(\cdot)$ are not related by a positive linear transformation. Because the gambling effect model satisfies EU when the set of options contains only risky lotteries, violations of EU can only occur at the corners of the probability triangle.

The gambling effect model can explain two well-known violations of EU, both due to Allais (1953). The *common consequence effect* refers to the empirical observation that there exist outcomes $a \succ b \succ c$ such that $S = (1, b)$ is preferred to $R = (p_1, a; p_2, b; 1-p_1-p_2, c)$, and $R' = (a, p_1; c)$ is preferred to $S' = (c, p_2; b)$. The *common ratio effect* refers to the observation that there exist outcomes $a \succ b \succ c$ such that for $\alpha \in (0, 1)$ $S = (1, b)$ is preferred to $R = (a, p_1; c)$, and $R' = (a, \alpha \cdot p_1; c)$ is preferred to $S' = (b, \alpha; c)$. The common consequence effect and the common ratio effect can be explained by the gambling effect model if $v(\cdot)$ is a concave transformation of u. Empirical evidence suggests that the elicited utility function is more concave when one of the lotteries is riskless than when both lotteries are risky (McCord and de Neufville 1986, Wakker and Deneffe 1996). In §6, we characterize the case in which $v(\cdot)$ is more concave than $u(\cdot)$.

The models by Fishburn (1980), Schmidt (1998), and Diecidue et al. (2001) also contain two separate utility functions, $u(\cdot)$ and $v(\cdot)$. Similar models were considered in Tversky (1967) and Conlisk (1993). In these models, $v(\cdot)$ is used to evaluate riskless lotteries and $u(\cdot)$ is used to evaluate all risky lotteries. Dyer and Sarin (1982) also suggested different evaluations for riskless and risky lotteries. In Dyer and Sarin's model, $u(\cdot)$ is used in evaluations where at least one lottery is risky. The utility function $v(\cdot)$ is only used in evaluations in which all lotteries are riskless. Dyer and Sarin's model is consistent with expected utility and therefore cannot explain the gambling effect. The function $v(\cdot)$ in Dyer and Sarin's model is different from our function $v(\cdot)$. In our model, $v(\cdot)$ is related to risky choice and is solely derived from revealed choice. Dyer and Sarin's function $v(\cdot)$ does not capture risk attitude and takes strength of preference as its primitive. Both Dyer and Sarin's function $v(\cdot)$ and our function $v(\cdot)$ represent preferences over outcomes. Hence, it appears plausible that our function $v(\cdot)$ is a strictly increasing transformation of Dyer and Sarin's function $v(\cdot)$. Our function $u(\cdot)$ and Dyer and Sarin's function $u(\cdot)$ both represent preferences over risky lotteries. Therefore, it is arguable that these two functions are also related by a strictly increasing transformation. Dyer and Sarin's functions $u(\cdot)$ and $v(\cdot)$ are related by a strictly increasing transformation because they both represent preferences over outcomes. The above argument, therefore, might suggest that our functions $u(\cdot)$ and $v(\cdot)$ are also related by a strictly increasing transformation. This is not necessarily true, however, which shows that our model is really different from Dyer and Sarin's model and that there is no clear relationship between our functions $u(\cdot)$ and $v(\cdot)$ and Dyer and Sarin's functions $u(\cdot)$ and $v(\cdot)$.

Table 1 summarizes the different models that employ separate utility functions $u(\cdot)$ and $v(\cdot)$.

Neilson (1992) and Humphrey (1998) proposed models of decision under risk where for each natural number n a utility function u_n is given. Lotteries with exactly n positive-probability outcomes are evaluated by expected utility with respect to u_n. The models of Fishburn (1980), Schmidt (1998), and Diecidue et al. (2001) are special cases of the Neilson-Humphrey model where only u_1 deviates from the other utilities and all u_js for $j \geq 2$ are equal. Schmidt (2001) showed that the basic idea of the Neilson-Humphrey model can also be integrated in the lottery-dependent utility model (Becker and Sarin 1987). In lottery-dependent utility the utility function depends on the lottery being evaluated, and lotteries with the same number of positive probability outcomes are generally evaluated by different utility functions.

In the health literature, Gafni et al. (1993) have suggested different utility functions $u(\cdot)$ and $v(\cdot)$. Their model is similar to Dyer and Sarin's (1982) model in that $v(\cdot)$ is only used if all lotteries are riskless. The function $u(\cdot)$ is used if at least one lottery is risky. Gafni et al.'s model differs from Dyer and Sarin's model in that $u(\cdot)$ and $v(\cdot)$ can order outcomes differently. Consequently, their model allows intransitivities (Loomes 1995, Wakker 1996). Gafni et al.'s model can neither explain the common consequence

Table 1 Overview of Models Using Different Utility Functions

Model	Two Riskless Lotteries $x \succcurlyeq y$	One Riskless and One Risky Lottery $x \succcurlyeq P$	Two Risky Lotteries $Q \succcurlyeq P$
Dyer and Sarin (1982)	$v(x) \geq v(y)$	$u(x) \geq \sum_{i=1}^{n} p_i \cdot u(x_i)$	$\sum_{i=1}^{n} q_i \cdot u(x_i) \geq \sum_{i=1}^{n} p_i \cdot u(x_i)$
Fishburn (1980) Schmidt (1998) Diecidue et al. (2001)	$v(x) \geq v(y)$	$v(x) \geq \sum_{i=1}^{n} p_i \cdot u(x_i)$	$\sum_{i=1}^{n} q_i \cdot u(x_i) \geq \sum_{i=1}^{n} p_i \cdot u(x_i)$
Present model	$v(x) \geq v(y)$	$v(x) \geq \sum_{i=1}^{n} p_i \cdot v(x_i)$	$\sum_{i=1}^{n} q_i \cdot u(x_i) \geq \sum_{i=1}^{n} p_i \cdot u(x_i)$

effect, nor the common ratio effect, nor the gambling effect. Gafni et al. proposed their model to defend the healthy-years-equivalents (HYE) measure (Mehrez and Gafni 1989) for the measurement of health. Because of the empirical implausibility of Gafni et al.'s utility model, the HYE has been largely discarded in the medical literature. This is unfortunate, because the HYE measure has valuable properties, in particular its focus on complete episodes of treatment. The gambling effect model can give a sound theoretical foundation to the HYE idea.

3. Characterization

Define two preference relations $\succcurlyeq_r$ and $\succcurlyeq_s$ from $\succcurlyeq$ as follows,

(1) for $P, Q \in \mathscr{R}$, $P \succcurlyeq_r Q$ if $P \succcurlyeq Q$;

(2) for P or $Q \in \mathscr{X}$, $P \succcurlyeq_s Q$ if $P \succcurlyeq Q$.

We derive expected utility representations for $\succcurlyeq_r$ and for $\succcurlyeq_s$. To do so, we reformulate Jensen's (1967) axioms in terms of $\succcurlyeq_r$ and $\succcurlyeq_s$. We impose the following assumptions on $\succcurlyeq_r$:

(i) *Weak order*: $\succcurlyeq_r$ is a weak order, i.e., it is transitive and complete.

(ii) *R-Independence*: For all $P, Q \in \mathscr{R}$, $R \in \mathscr{P}$, $\lambda \in (0, 1]$, if $P \succcurlyeq_r Q$ then $\lambda P + (1-\lambda)R \succcurlyeq_r \lambda Q + (1-\lambda)R$.

(iii) *R-Continuity:* For all $P, Q, R \in \mathscr{R}$, if $P \succ_r Q \succ_r R$, then there exist $\lambda, \mu \in (0, 1)$ such that $\lambda P + (1-\lambda)R \succ_r Q$ and $Q \succ_r \mu P + (1-\mu)R$.

R-independence is a weakening of von Neumann-Morgenstern independence to preference comparisons between risky lotteries.

We impose three axioms on $\succcurlyeq_s$.

(iv) *Weak order*: (i) For all $P, Q \in \mathscr{P}$ with P or $Q \in \mathscr{X}$, $P \succcurlyeq_s Q$ or $Q \succcurlyeq_s P$, and (ii) for all $P, Q, R \in \mathscr{P}$ with P or $R \in \mathscr{X}$, if $P \succcurlyeq_s Q$ and $Q \succcurlyeq_s R$, then $P \succcurlyeq_s R$.

Formulating independence is somewhat difficult. The common definition of independence involves preference comparison between probabilistic mixtures, i.e., between two elements of $\mathscr{R}$ and, hence, provides no information on $\succcurlyeq_s$. To compare probability mixtures we must use certainty equivalents as intermediaries. We now define independence for $\succcurlyeq_s$.

(v) *S-Independence*: For all $P, Q, R \in \mathscr{P}$, $x, y, x', y' \in \mathscr{X}$, $\lambda \in [0, 1]$, if $P \sim_s x$, $Q \sim_s y$, $\lambda P + (1-\lambda)R \sim_s x'$, $\lambda Q + (1-\lambda)R \sim_s y'$, and $x \succcurlyeq_s y$, then $x' \succcurlyeq_s y'$.

S-Independence says that if a certainty equivalent of P is weakly preferred to a certainty equivalent of Q, then a certainty equivalent of any probabilistic mixture of P with R should be weakly preferred to a certainty equivalent of the same probabilistic mixture of Q with R.

Our final condition is a continuity condition which requires that for every outcome that is not weakly preferred or dispreferred to all lotteries there exists a risky lottery that is equivalent to it.

(vi) *S-continuity*: For all $x \in \mathscr{X}$, $P, Q \in \mathscr{P}$, if $P \succ_s x \succ_s Q$, then there exists a $\lambda \in (0, 1)$ such that $\lambda P + (1-\lambda)Q \sim_s x$.

THEOREM 3.1. *The following two statements are equivalent*:

(i) $\succcurlyeq$ *can be represented by the gambling effect model*.

(ii) $\succcurlyeq$ *satisfies axioms* (*i*)–(*vi*).

4. Stochastic Dominance and Transitivity

Stochastic Dominance

The analysis of the preceding section did not impose any restrictions on the relation between $u(\cdot)$ and $v(\cdot)$. We now impose a condition that ensures that $u(\cdot)$ and $v(\cdot)$ are *ordinally equivalent*, i.e., for all $x, y \in \mathscr{X}$, $u(x) \geq u(y)$ iff $v(x) \geq v(y)$. It is well known that ordinal equivalence of $u(\cdot)$ and $v(\cdot)$ implies that there exists a strictly increasing function f such that for all $x \in \mathscr{X}$, $v(x) = f(u(x))$.

The preference relation $\succcurlyeq$ satisfies *gamble monotonicity* if for all $R \in \mathscr{R}$, for all $x, y \in \mathscr{X}$, and for all $\lambda \in (0, 1]$, $x \succcurlyeq y$ iff $\lambda x + (1-\lambda)R \succcurlyeq \lambda y + (1-\lambda)R$. Gamble monotonicity was introduced by Diecidue et al. (2001) and says that the replacement of an outcome by a preferred outcome leads to a preferred lottery. R is a risky lottery in the definition of gamble monotonicity to ensure that the gambles $\lambda x + (1-\lambda)R$ and $\lambda y + (1-\lambda)R$ are both risky and are evaluated by u.

THEOREM 4.1. *Suppose that* $\succcurlyeq$ *can be represented by the gambling effect model. The following two statements are equivalent*:

(i) $u(\cdot)$ *and* $v(\cdot)$ *are ordinally equivalent*.

(ii) $\succcurlyeq$ *satisfies gamble monotonicity*.

The preference relation $\succcurlyeq$ satisfies *stochastic dominance* if for all $R \in \mathscr{P}$, for all $x, y \in \mathscr{X}$, and for all $\lambda \in (0, 1]$, if $x \succcurlyeq y$, then $\lambda x + (1-\lambda)R \succcurlyeq \lambda y + (1-\lambda)R$. The main difference between stochastic dominance and gamble monotonicity is that in the definition of stochastic dominance all lotteries R are allowed, whereas in the definition of gamble monotonicity R has to be risky.

Corollary 4.2. *Suppose that $\succcurlyeq$ can be represented by the gambling effect model. If $u(\cdot)$ and $v(\cdot)$ are ordinally equivalent, then $\succcurlyeq$ satisfies stochastic dominance.*

Transitivity

The gambling effect model can violate transitivity. For example, let $v(x) = \sqrt{x}$, $u(x) = x$, $P = (100, 0.5; 0)$, $Q = (81, 0.5; 9)$, and $x = 30$. Then $P \succ Q$ and $Q \succ x$, but $x \succ P$. The next theorem shows that the exclusion of intransitivities implies that $u(\cdot)$ and $v(\cdot)$ can be chosen equal, i.e., that EU holds. To the extent that transitivity is normative, the theorem limits the prescriptive applicability of the gambling effect model. It does not necessarily limit the descriptive applicability of the model because violations of transitivity that are consistent with our model are commonly observed in empirical research.

Theorem 4.3. *Suppose that $\succcurlyeq$ can be represented by the gambling effect model. The following two statements are equivalent*:

(i) *$\succcurlyeq$ satisfies transitivity and $u(\cdot)$ and $v(\cdot)$ are ordinally equivalent*.

(ii) *EU holds*.

5. Elicitation

Let $M, m \in \mathscr{X}$, $M \succ m$. Set $u(M) = 1$ and $u(m) = 0$. Let $x \in \mathscr{X}$, $x \neq M$, $x \neq m$, and fix $\lambda \in (0, 1)$. The function $u(\cdot)$ can be elicited through a lottery equivalence procedure (McCord and de Neufville 1986). The elicitation procedure is described in Table 2, where μ is the parameter that must be elicited. The first row of the table shows the question that must be asked; the second shows the resulting utility.

Table 2 **The Elicitation of $u(\cdot)$**

Case	$(M, \lambda; x) \succcurlyeq (M, \lambda; m)$	$(M, \lambda; x) \prec (M, \lambda; m)$
Question	$(M, \lambda; m) \sim (x, \mu; m)$	$(M, \lambda; m) \sim (M, \mu; x)$
$u(x)$	λ/μ	$(\lambda - \mu)/(1-\mu)$

Obviously, it is recommended to use easily perceived values of λ in the elicitation of $u(\cdot)$, e.g. $\lambda = 1/2$. The function $v(\cdot)$ can be elicited through standard probability equivalence or certainty equivalence questions (Farquhar 1984).

6. Certainty Preference

The empirical literature suggests that $v(\cdot)$ is more concave than $u(\cdot)$. We now characterize this case. Throughout this section, we assume that $\mathscr{X}$ is a connected topological space. An outcome $y_p \in \mathscr{X}$ is a *conditional certainty equivalent* of a lottery $P \in \mathscr{P}$ if there exists a $\lambda \in (0, 1)$ and a lottery $R \in \mathscr{R}$ such that $\lambda y_p + (1-\lambda)R \sim \lambda P + (1-\lambda)R$. The gambling effect model implies that for $R \in \mathscr{R}$ if for some $\lambda \in (0, 1)$, $\lambda y_p + (1-\lambda)R \sim \lambda P + (1-\lambda)R$, then for all $\lambda \in (0, 1)$, $\lambda y_p + (1-\lambda)R \sim \lambda P + (1-\lambda)R$. The preference relation $\succcurlyeq$ satisfies *gambling aversion* if for all $P \in \mathscr{P}$, $y_p \succcurlyeq x_p$, where x_p is the certainty equivalent of P. The function $v(\cdot)$ is a *concave transformation* of $u(\cdot)$ if $v = f(u)$ with $f(\cdot)$ strictly increasing and concave, i.e., for all $x \in \mathscr{X}$ $f'(x) > 0$ and $f''(x) < 0$.

Theorem 6.1. *Suppose that $\succcurlyeq$ can be represented by the gambling effect model with $u(\cdot)$ and $v(\cdot)$ ordinally equivalent and continuous. The following two statements are equivalent*:

(i) *$\succcurlyeq$ satisfies gambling aversion*.

(ii) *$v(\cdot)$ is a concave transformation of $u(\cdot)$*.

7. Risk Aversion

The use of separate functions $u(\cdot)$ and $v(\cdot)$ allows the decomposition of attitudes towards gambling and "pure" attitudes towards risk. Attitudes towards gambling apply to comparisons where one lottery is riskless. Pure attitudes towards risk, i.e., attitudes that are not confounded by the gambling effect, apply to comparisons between risky lotteries. To consider risk attitudes, we assume that $\mathscr{X}$ is a compact interval of the real line, that $u(\cdot)$ and $v(\cdot)$ are increasing in x, and are twice differentiable.

Mean-Preserving Spreads
Theorem 7.1 shows that aversion to mean-preserving spreads, which in expected utility is equivalent to concavity of the utility function, corresponds in the gambling effect model to concavity of both $u(\cdot)$ and $v(\cdot)$.

THEOREM 7.1. *Suppose that $\succcurlyeq$ can be represented by the gambling effect model. The following statements are equivalent*:

(i) *For all $P \in \mathcal{P}$, $Q \in \mathcal{R}$, if Q is a mean-preserving spread of P, then $P \succcurlyeq Q$.*

(ii) *$u(\cdot)$ and $v(\cdot)$ are concave.*

The Risk Premium and the Gambling Premium
The interpretation of the risk premium of a lottery P, defined as $e_p - x_p$ with e_p and x_p the expected value and the certainty equivalent of P, respectively, is ambiguous in the gambling effect model, because $P \sim x_p$ does not imply $u(P) = u(x_p)$ unless $u(\cdot) = v(\cdot)$. To resolve this ambiguity, we split the risk premium into two separate premiums, the *conditional risk premium* $e_p - y_p$ and the *gambling premium* $y_p - x_p$, where y_p is the conditional certainty equivalent of P. Clearly, the risk premium is equal to the sum of the conditional risk premium and the gambling premium. The gambling premium is equal to zero for all $P \in \mathcal{P}$ if and only if EU holds and y_p is equal to x_p. Because $v(\cdot)$ is strictly increasing, the gambling premium is nonnegative if and only if $\succcurlyeq$ satisfies gambling aversion.

Comparative Risk Aversion and Gambling Aversion
In the gambling effect model it makes sense to say that the Arrow-Pratt measure $-u''(x)/u'(x)$ captures "pure" risk aversion, i.e., the risk aversion that is observed in comparisons between risky lotteries. Theorem 6.1 suggests that we can define $-f''(u)/f'(u)$ as a measure of comparative gambling aversion. Let $\succcurlyeq_i$ denote the preference relation of individual i, $i = 1, 2$. For $P \in \mathcal{P}$, let $x_{i,p}$ denote individual i's certainty equivalent of lottery P. $x_{i,p}$ is uniquely defined because $v_i(\cdot)$ is strictly increasing. The preference relation $\succcurlyeq_1$ is *more gambling averse* than the preference relation $\succcurlyeq_2$ if for all $x \in \mathcal{X}$, $P, Q \in \mathcal{R}$, $R \in \mathcal{P}$, $\lambda \in (0,1)$ such that $\lambda x + (1-\lambda)R \sim_1 \lambda P + (1-\lambda)R$ and $\lambda x + (1-\lambda)R \sim_2 \lambda Q + (1-\lambda)R$, $x_{1,p} \leq x_{2,q}$. That is, if Individual 1's conditional certainty equivalent of P equals Individual 2's conditional certainty equivalent of Q, then Individual 1 is more gambling averse than Individual 2 if his certainty equivalent of P does not exceed Individual 2's certainty equivalent of Q.

THEOREM 7.2. *Let $v_i = f_i(u_i)$, $i = 1, 2$. Suppose that $\succcurlyeq_1$ and $\succcurlyeq_2$ can be represented by the gambling effect model. The following two statements are equivalent*:

(i) *$\succcurlyeq_1$ is more gambling averse than $\succcurlyeq_2$.*

(ii) *For all $u \in \mathbb{R}$,*

$$-\frac{f_1''(u)}{f_1'(u)} \geq -\frac{f_2''(u)}{f_2'(u)}.$$

Individual 1 is *more risk averse than* Individual 2 if for every risky lottery P Individual 1 assigns neither a higher certainty equivalent nor a higher conditional certainty equivalent to P than Individual 2. Formally, for all $P \in \mathcal{R}$, $Q \in \mathcal{P}$, $\lambda \in (0, 1]$ if $\lambda y_{1,p} + (1-\lambda)Q \sim_1 \lambda P + (1-\lambda)Q$ and $\lambda y_{2,p} + (1-\lambda)Q \sim_2 \lambda P + (1-\lambda)Q$, then $y_{1,p} \leq y_{2,p}$.

THEOREM 7.3. *Suppose that $\succcurlyeq_1$ and $\succcurlyeq_2$ can be represented by the gambling effect model. The following two statements are equivalent*:

(i) *$\succcurlyeq_1$ is more risk averse than $\succcurlyeq_2$.*

(ii) *for all $x \in \mathcal{X}$,*

$$-\frac{u_1''(x)}{u_1'(x)} \geq -\frac{u_2''(x)}{u_2'(x)}$$

and for all $u \in \mathbb{R}$,

$$-\frac{f_1''(u)}{f_1'(u)} \geq -\frac{f_2''(u)}{f_2'(u)}.$$

8. Conclusion

To generate fruitful applications, a decision model should satisfy three requirements: It should be intuitively appealing, descriptively accurate, and applicable in practice. We believe that the gambling effect has intuitive appeal, being based on the notion that the presence of riskless lotteries triggers different emotions and changes people's evaluation process. In general, there is a trade-off between the requirements of descriptive accuracy and practical applicability. Descriptive accuracy requires the model to account for many preference patterns. Practical applicability limits the number of preference patterns the model can explain, because the complexity of the elicitation task generally increases with the number of patterns consistent with the model.

Harless and Camerer (1994) conclude concerning the trade-off between descriptive accuracy and parsimony that: "expected utility [is] too lean: [it] could explain the data better by allowing a few more common patterns. Other theories, such as mixed fanning and rank-dependent expected utility, are too fat: They allow a lot of patterns which are rarely observed" (p. 1285). The gambling effect model deviates minimally from expected utility. The violations of expected utility that are consistent with our model are frequently observed. For example, the most common violations of expected utility in Loomes et al. (1991) and in Sopher and Gigliotti (1993) are violations allowed by the gambling effect model. There are several empirically observed violations of EU that cannot be explained by our model, for example, frequently observed violations of expected utility on the boundaries of the probability triangle that involve no riskless lotteries. The extent to which such violations are problematic for our theory, indicating that our theory is too lean, depends on how systematic these violations are. The literature gives no clear answer to this question. For example, regarding violations of EU at the boundaries of the probability triangle, Starmer and Sugden (1989) conclude that: "the most striking feature of our results is the absence of any obvious general pattern to the violation of EU" (p. 99).

The primary use of our theory is for the decision analyst who wants a model that captures the main deviations from expected utility, but that is applicable in practice. We hope that the gambling effect model provides him with such a model.

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Appendix: Proofs

Proof of Theorem 3.1. It is straightforward to verify that the gambling effect model implies conditions (i)–(vi). Let us therefore assume conditions (i)–(vi) and derive the gambling effect model. Fishburn (1980) and Diecidue et al. (2001) proved that $\succcurlyeq_r$ has an EU representation. It remains to prove that $\succcurlyeq_s$ has an EU representation.

Lemma 1. *For all $P, Q \in \mathcal{P}$, λ, $\mu \in [0,1]$, if $x \sim_s P$, $y \sim_s Q$, $x' \sim_s \lambda P + (1-\lambda)Q$, $y' \sim_s \mu P + (1-\mu)Q$, and $x \succ_s y$, then $x' \succ_s y'$ iff $\lambda > \mu$.*

Proof. Let $x \sim_s P$, $y \sim_s Q$, $x' \sim_s \lambda P + (1-\lambda)Q$, $y' \sim_s \mu P + (1-\mu)Q$, and $x \succ_s y$. Suppose that $\lambda > \mu$. Consider $\lambda P + (1-\lambda)Q = (\lambda-\mu)P + (1-(\lambda-\mu))R$ and $\mu P + (1-\mu)Q = (\lambda-\mu)Q + (1-(\lambda-\mu))R$ with

$$R = \left(\frac{\mu}{1-(\lambda-\mu)}P + \frac{1-\lambda}{1-(\lambda-\mu)}Q\right) \in \mathcal{P}.$$

S-independence implies that $x' \succ_s y'$.

Suppose now that $x' \succ_s y'$. Suppose that $\mu \geq \lambda$. If $\mu = \lambda$, then $\lambda P + (1-\lambda)Q = \mu P + (1-\mu)Q$, and it follows by transitivity that $x' \sim_s y'$, a contradiction. Suppose, therefore, that $\mu > \lambda$. Consider $\lambda P + (1-\lambda)Q = (\mu-\lambda)Q + (1-(\mu-\lambda))R'$ and $\mu P + (1-\mu)Q = (\mu-\lambda)P + (1-(\mu-\lambda))R'$ with

$$R' = \left(\frac{\lambda}{1-(\mu-\lambda)}P + \frac{1-\mu}{1-(\mu-\lambda)}Q\right) \in \mathcal{P}.$$

S-independence implies that $y' \succ_s x'$, a contradiction. □

Let $x \sim_s P$, $y \sim_s Q$, $x \succ_s z \succ_s y$. By S-continuity there exists a $\lambda \in (0,1)$ such that $z \sim_s \lambda P + (1-\lambda)Q$. Lemma 1 implies that λ is unique. For suppose that both $z \sim_s \lambda_1 P + (1-\lambda_1)Q$ and $z \sim_s \lambda_2 P + (1-\lambda_2)Q$ with $\lambda_1 > \lambda_2$. Then Lemma 1 implies that $z \succ_s z$, a contradiction. Similarly $\lambda_2 > \lambda_1$ leads to a contradiction. Hence, $\lambda_1 = \lambda_2$ and λ is unique.

Lemma 2. *For all $P, Q, R \in \mathcal{P}$, $\lambda \in [0,1]$, if $x \sim_s P$, $y \sim_s Q$, $x' \sim_s \lambda P + (1-\lambda)R$, $y' \sim_s \lambda Q + (1-\lambda)R$, and $x \sim_s y$, then $x' \sim_s y'$.*

Proof. By S-independence, we have $x' \succcurlyeq_s y'$. Suppose that $x' \succ_s y'$. $P = 0\cdot(\lambda P + (1-\lambda)R) + P = 0\cdot(\lambda Q + (1-\lambda)R) + P$. Because $x \sim_s P$, it follows from $x' \sim_s \lambda P + (1-\lambda)R$, $y' \sim_s \lambda Q + (1-\lambda)R$, and $x' \succ y'$ by S-independence that $x \succ_s x$, which contradicts weak ordering of $\succ_s$. Hence, $x' \sim_s y'$. □

We now define the utility function. Suppose first that the set $\mathcal{X}$ contains a best outcome M and a worst outcome m. That is, for all $x \in \mathcal{X}$, $M \succcurlyeq_s x$, and $x \succcurlyeq_s m$. This assumption will be relaxed shortly. Define for all $x \in \mathcal{X}$, $v(x) = \lambda$ where λ is the unique number such that $x \sim_s \lambda M + (1-\lambda)m$. Obviously, this implies that $v(M) = 1$ and $v(m) = 0$. If $v(x) = \lambda > v(y) = \mu$, then we have $x \sim_s \lambda M + (1-\lambda)m$ and $y \sim_s \mu M + (1-\mu)m$ and Lemma 1 implies that $x \succ_s y$. Lemma 1 also implies that if $x \succ_s y$, then $v(x) = \lambda > v(y) = \mu$. If $x \sim_s y$, then $v(x) = v(y)$ because λ is uniquely determined. If $v(x) = v(y) = \lambda$, then $x \sim_s \lambda M + (1-\lambda)m \sim_s y$ and by transitivity $x \sim_s y$. Hence, for all $x, y \in \mathcal{X}$, $x \succcurlyeq_s y$ iff $v(x) \geq v(y)$. For any lottery $P \in \mathcal{P}$, there is an outcome $z \in \mathscr{X}$ such that $z \sim_s P$ by assumption. Define $v(P) = v(z)$. Then $v(\cdot)$ preserves $\succcurlyeq_s$ on $\mathscr{P}$.

We next prove that $v(\cdot)$ is linear, i.e., $v(\sum_{i=1}^n p_i x_i) = \sum_{i=1}^n p_i v(x_i)$. Denote $P = \sum_{i=1}^n p_i x_i$ and let $z \in \mathscr{X}$ be such that $z \sim_s P$. Construct the lottery $P^{(1)}$ by replacing x_1 in P by the lottery $\lambda_1 M + (1-\lambda_1)m \sim_s x_1$. By Lemma 2 and transitivity, $z \sim_s P^{(1)}$. Proceed by constructing the lottery $P^{(i)}$ from $P^{(i-1)}$, $i = 2, \ldots, n$, by replacing x_i by $\lambda_i M + (1-\lambda_i)m \sim_s x_i$. By Lemma 2 and transitivity we have $z \sim_s P^{(i)}$, $i = 2, \ldots, n$. Now $P^{(n)} = \sum_{i=1}^n p_i \lambda_i M + (1 - \sum_{i=1}^n p_i \lambda_i)m$. Hence $P \sim_s z \sim_s P^{(n)}$ implies $v(P) = v(z) = \sum_{i=1}^n p_i \lambda_i = \sum_{i=1}^n p_i v(x_i)$. Hence, $v(\cdot)$ is linear.

Suppose now that $\mathscr{X}$ contains no best and worst outcomes with respect to $\succcurlyeq_s$. Then for each lottery P there are outcomes $x, y \in \mathscr{X}$ such that $x \succ_s P \succ_s y$. Let $\mathscr{P}_{xy}$ denote the set $\{P \in \mathscr{P}\colon x \succ_s P \succ_s y\}$. For all $z \in \mathscr{X} \cap \mathscr{P}_{xy}$ define $v_{xy}(z) = \lambda$ if $z \sim_s \lambda x + (1-\lambda)y$. For all $P \in \mathscr{P}_{xy}$ define $v(P) = v(z)$ if $z \sim_s P$. The proof that v_{xy} is order preserving and linear on $\mathscr{P}_{xy}$ is as in the case above where $\mathscr{X}$ contains a best and a worst outcome.

The rest of the proof is identical to Jensen (1967). Fix outcomes $R, r \in \mathscr{X}$ such that $x \succ_s R \succ_s r \succ_s y$. For any $P \in \mathscr{P}_{xy}$ define

$$H_{xy}(P) = \frac{v_{xy}(P) - v(r)}{v(R) - v(r)}.$$

H_{xy} is order preserving and linear because v_{xy} is. Two functions H_{xy} and H_{vw} are identical on common domain. Hence, we can define for all $P \in \mathscr{P}$, $v(P) = H_{xy}(P)$ where the choice of x and y is immaterial. $v(\cdot)$ is bounded and unique up to positive linear transformations. □

Proof of Theorem 4.1. Let $R \in \mathscr{R}$. Then for all $x, y \in \mathscr{X}, \lambda \in (0,1)$, the lotteries $\lambda x + (1-\lambda)R$ and $\lambda y + (1-\lambda)R$ are both risky. Suppose first that $u(\cdot)$ and $v(\cdot)$ are ordinally equivalent. Let $x, y \in \mathscr{X}$. If $x \succcurlyeq y$, then $v(x) \geq v(y)$ by the gambling effect model. Because $u(\cdot)$ and $v(\cdot)$ are ordinally equivalent, also $u(x) \geq u(y)$. Hence, by the gambling effect model $\lambda x + (1-\lambda)R \succcurlyeq \lambda y + (1-\lambda)R$. Suppose next that $\lambda x + (1-\lambda)R \succcurlyeq \lambda y + (1-\lambda)R$. Then $u(x) \geq u(y)$ by the gambling effect model, $v(x) \geq v(y)$ by ordinal equivalence of $u(\cdot)$ and $v(\cdot)$, and $x \succcurlyeq y$ by the gambling effect model.

Suppose now that gamble monotonicity holds. Let $x, y \in \mathscr{X}$. If $v(x) \geq v(y)$, then $x \succcurlyeq y$ by the gambling effect model. Hence, $\lambda x + (1-\lambda)R \succcurlyeq \lambda y + (1-\lambda)R$ by gamble monotonicity, and $u(x) \geq u(y)$ by the gambling effect model. If $u(x) \geq u(y)$, then $\lambda x + (1-\lambda)R \succcurlyeq \lambda y + (1-\lambda)R$ by the gambling effect model, $x \succcurlyeq y$ by gamble monotonicity, and $v(x) \geq v(y)$ by the gambling effect model. □

Proof of Corollary 4.2. Let $x, y \in \mathscr{X}, x \succcurlyeq y$. Then $v(x) \geq v(y)$ by the gambling effect model. If R is risky, then the proof that $\lambda x + (1-\lambda)R \succcurlyeq \lambda y + (1-\lambda)R$ follows from Theorem 4.1. If R is riskless but different from x and y, then $\lambda x + (1-\lambda)R \succcurlyeq \lambda y + (1-\lambda)R$ iff $u(x) \geq u(y)$, which holds by ordinal equivalence. If R is riskless and equal to x or y, then either $\lambda x + (1-\lambda)R$ or $\lambda y + (1-\lambda)R$ is riskless, and hence the preference comparison is made in terms of $\succcurlyeq_s$. Because this preference relation can be represented by an EU functional, stochastic dominance is satisfied and thus $\lambda x + (1-\lambda)R \succcurlyeq \lambda y + (1-\lambda)R$. □

Proof of Theorem 4.3. It is well known that EU satisfies transitivity. Hence, we assume transitivity and the gambling effect model and derive EU. Let $P, Q \in \mathscr{R}$ and let $x_p \sim P, x_q \sim Q$. Now $\sum_{i=1}^n p_i u(x_i) > \sum_{i=1}^n q_i u(x_i)$ implies $P \succ Q$ by the gambling effect model. By transitivity, $x_p \succ x_q$, and by the gambling effect model, $\sum_{i=1}^n p_i v(x_i) > \sum_{i=1}^n q_i v(x_i)$. If $\sum_{i=1}^n p_i v(x_i) > \sum_{i=1}^n q_i v(x_i)$, then $x_p \succ x_q$ by the gambling effect model, $P \succ Q$ by transitivity, and $\sum_{i=1}^n p_i u(x_i) > \sum_{i=1}^n q_i u(x_i)$ by the gambling effect model. Thus, $\sum_{i=1}^n p_i u(x_i) > \sum_{i=1}^n q_i u(x_i)$ iff $\sum_{i=1}^n p_i v(x_i) > \sum_{i=1}^n q_i v(x_i)$. It can be shown similarly that $\sum_{i=1}^n p_i u(x_i) = \sum_{i=1}^n q_i u(x_i)$ iff $\sum_{i=1}^n p_i v(x_i) = \sum_{i=1}^n q_i v(x_i)$ and that $\sum_{i=1}^n p_i u(x_i) < \sum_{i=1}^n q_i u(x_i)$ iff $\sum_{i=1}^n p_i v(x_i) < \sum_{i=1}^n q_i v(x_i)$. Thus, $u(\cdot)$ and $v(\cdot)$ are cardinally equivalent and can be chosen identical. □

Proof of Theorem 6.1. Because $\mathscr{X}$ is a connected topological space and $u(\cdot)$ is continuous, each lottery $P \in \mathscr{P}$ has at least one conditional certainty equivalent. Pratt (1964, Theorem 1) has shown for real outcomes and $u(\cdot)$ and $v(\cdot)$ increasing and continuous that for all $P \in \mathscr{P}$, $v(\cdot)$ is a concave transformation of $u(\cdot)$ iff the certainty equivalent of P with respect to $u(\cdot)$ is preferred to the certainty equivalent of P with respect to $v(\cdot)$. Remark VII.6.6. in Wakker (1989) generalizes Pratt's result to connected topological spaces and $u(\cdot)$ and $v(\cdot)$ ordinally equivalent and continuous. □

Proof of Theorem 7.1. Let Q be a mean-preserving spread of P. By Theorem 2 in Rothschild and Stiglitz (1970), if $P \in \mathscr{R}$, then $P \succcurlyeq Q$ iff $u(\cdot)$ is concave; if $P \in \mathscr{X}$, then $P \succcurlyeq Q$ iff $v(\cdot)$ is concave. □

Proof of Theorem 7.2. Suppose that for some $x \in \mathscr{X}, P, Q \in \mathscr{R}$, $R \in \mathscr{P}, \lambda \in (0,1)$, $\lambda x + (1-\lambda)R \sim_1 \lambda P + (1-\lambda)R$ and $\lambda x + (1-\lambda)R \sim_2 \lambda Q + (1-\lambda)R$. By the gambling effect model, $u_1(x) = \Sigma_i p_i \cdot u_1(x_i)$, $u_2(x) = \Sigma_i q_i \cdot u_2(x_i)$, $f_1(u_1(x_{1,p})) = \Sigma_i p_i \cdot f_1(u_1(x_i))$, and $f_2(u_2(x_{2,q})) = \Sigma_i q_i \cdot f_2(u_2(x_i))$. It now follows immediately from Theorem 1 in Pratt (1964) and f and $u(\cdot)$ increasing that $x_{1,p} \leq x_{2,q}$ iff $f_1(\cdot)$ is a concave transformation of $f_2(\cdot)$, i.e., iff for all $u \in \mathbb{R}$,

$$-\frac{f_1''(u)}{f_1'(u)} \geq -\frac{f_2''(u)}{f_2'(u)}. \quad \square$$

Proof of Theorem 7.3. Suppose first that (ii) holds. If $\lambda \in (0,1)$, then (i) follows from Theorem 1 in Pratt (1964). Let $\lambda = 1$. The combination of for all $x \in \mathscr{X}$,

$$-\frac{u_1''(x)}{u_1'(x)} \geq -\frac{u_2''(x)}{u_2'(x)}$$

and for all $u \in \mathbb{R}$,

$$-\frac{f_1''(u)}{f_1'(u)} \geq -\frac{f_2''(u)}{f_2'(u)}$$

implies that for all $x \in \mathscr{X}$,

$$-\frac{v_1''(x)}{v_1'(x)} \geq -\frac{v_2''(x)}{v_2'(x)}.$$

Statement (i) follows by Theorem 1 in Pratt (1964).

Suppose now that (i) holds. If $\lambda \in (0,1)$, then Theorem 1 in Pratt (1964) implies that for all $x \in \mathscr{X}$,

$$-\frac{u_1''(x)}{u_1'(x)} \geq -\frac{u_2''(x)}{u_2'(x)},$$

i.e., Individual 1 has a higher Arrow-Pratt measure of absolute risk aversion than Individual 2. Let $\lambda = 1$. Suppose, contrary to statement (ii), that for some $u \in \mathscr{X}$,

$$-\frac{f_1''(u)}{f_1'(u)} < -\frac{f_2''(u)}{f_2'(u)}.$$

Fix $x \in \mathscr{X}$ and let $u_1(x) = u_2(x) = u$ which is allowed by the uniqueness properties of u_1 and u_2. Take an arbitrary $\varepsilon > 0$. Because u_1 and u_2 are continuous there exist δ_1 and δ_2 such that for $i = 1, 2$ if $|y - x| < \delta_i$, then $|u_i(y) - u_i(x)| < \varepsilon$. Define $\delta = \min\{\delta_1, \delta_2\}$. Let P be the lottery $(\frac{1}{2}, x + \delta; \frac{1}{2}, x - \delta)$. Because u_i is increasing, $i = 1, 2$, $\mathrm{EU}_i(P) = u_i(x) + \varepsilon$. It follows that if $\delta \to 0$, the conditional certainty equivalent of P goes to x both for Individual 1 and for Individual 2. Theorem 1 in Pratt (1964) implies a strict version of Theorem 7.2, which says that, if

$$-\frac{f_1''(u)}{f_1'(u)} < -\frac{f_2''(u)}{f_2'(u)},$$

then Individual 1's certainty equivalent of P is higher than that of Individual 2. This contradicts that Individual 1 is more risk averse than Individual 2. □

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