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Time preference, the discounted utility model and health

Han Bleichrodt [a,*], Amiram Gafni [b]

[a] *Institute for Medical Technology Assessment, Department of Health Policy and Management, Erasmus University, Rotterdam, The Netherlands*

[b] *Centre for Health Economics and Policy Analysis, Department of Clinical Epidemiology and Biostatistics, McMaster University, Hamilton, Ontario, Canada*

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Abstract

The constant rate discounted utility model is commonly used to represent intertemporal preferences in health care program evaluations. This paper examines the appropriateness of this model, and argues that the model fails both normatively and descriptively as a representation of individual intertemporal preferences for health outcomes. Variable rate discounted utility models are more flexible, but still require restrictive assumptions and may lead to dynamically inconsistent behaviour. The paper concludes by considering two ways of incorporating individual intertemporal preferences in health care program evaluations that allow for complementarity of health outcomes in different time periods.

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1. Introduction

Constant rate discounted utility models are commonly used to represent individual intertemporal preferences in health care program evaluation. The debate mainly centers around the question of what rate of discount to use. Little attention

* Corresponding author.

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has been paid to the appropriateness of the constant rate discounted utility model as such. The axioms underlying the individual preference structure to fit impatience (Koopmans, 1960), time perspective (Koopmans et al., 1964) and the discounted utility model both for a single outcome (Fishburn and Rubinstein, 1982) and for (infinite) sequences of outcomes (Koopmans, 1972) can be found in the economic literature. The general impression from this literature is that the discounted utility model [1] is far from realistic. This impression has been confirmed by empirical studies concerning time preference. These studies display a number of anomalies, that are robust and do not require ingenious experimental designs to be revealed. [2]

This paper examines the appropriateness of the discounted utility model as a description of an individual's intertemporal preferences for health outcomes. The analysis has immediate relevance for the appropriateness of the use of the discounted utility model in the context of economic evaluations where the social discount rate is assumed to be based on the aggregate of individuals' intertemporal preferences (e.g., Redelmeier and Heller, 1993; Weinstein, 1993). The conditions that the model imposes on the individual preference structure are derived and their restrictiveness is assessed. Both the case where the preference relation is defined over health outcomes and the case where the preference relation is defined over lotteries over health outcomes are addressed. We argue that in neither situation does the discounted utility model provide a good description of an individual's intertemporal preferences for health outcomes. The argument that the discounted utility model may not hold descriptively, but should be adopted because of its normative appeal will be considered but ultimately rejected.

It has been argued that the rejection of a constant rate of discount calls for the use of a model with a discount rate that is variable (see for example Olsen, 1993b). By examining the axiomatic structure of the model and by means of an example, we show that using a variable rate discounted utility model does not solve all problems of the constant rate discounted utility model and creates a problem of its own: it may entice the individual to behave in a dynamically inconsistent way.

In this paper we are concerned mainly with *individual* intertemporal preferences for sequences of health outcomes. One might argue that an individual's intertemporal preferences are of no interest in health care program evaluations given that health care program evaluation should be based on an appropriately selected social rate of discount. But when the social discount rate is to be based on the aggregate of the individual intertemporal preferences, as has often been

[1] From now on discounted utility model will stand for constant-rate discounted utility model unless otherwise stated.

[2] See for example Loewenstein (1987, 1988), Loewenstein and Prelec (1991, 1992, 1993), Loewenstein and Thaler (1989), and Thaler (1981). For examples of violations of the DU model with health outcomes see Olsen (1993a), and Redelmeier and Heller (1993).

advocated in the case of program evaluation, this argument runs into problems. It is not clear why an aggregate concept should satisfy a model that is violated by its constituent parts. On the other hand defining the social rate of discount without taking into account the individual's intertemporal preferences raises the question – what should the foundation of the social rate of discount be? It has been argued that one should select the appropriate market rate of interest corrected for tax distortions. However, correcting for tax distortions is far from straightforward and the relationship between the market rate of interest and the social rate of time preference is further distorted by the internationalisation of capital markets (Lind, 1990). Also, ignoring individual intertemporal preferences might be undesirable for reasons of consistency. Considerable attention is being given to the development of methods to elicit individuals' preferences for health outcomes. Because health outcomes have a time dimension inextricably bound to them, we cannot ignore individual intertemporal preferences in valuing them.

The structure of the paper is as follows. Section 2 derives the discounted utility model when the preference relation is defined over health outcomes under certainty. Section 3 derives the discounted utility model when the preference relation is defined over lotteries on health outcomes (i.e., under risk). Both sections are technical. In Section 4 the axioms underlying the discounted utility model are discussed from a normative point of view. Section 5 presents descriptive evidence concerning factors affecting individual intertemporal preferences. In Section 6 we discuss the argument that a variable rate discounted utility model should be used to model individual intertemporal preferences for health. Section 7 contains concluding remarks and considers two alternative approaches to incorporate individual intertemporal preferences in the evaluation of health care programs. Proofs of the various results presented in the paper appear in the appendices.

2. Intertemporal preferences under certainty

2.1. Preliminaries

This subsection introduces notation and structural assumptions. For more details the reader is referred to the appendices. The paper deals with an individual decision maker who has a preference relation $\succeq$, meaning "at least as preferred as", over a set X of health profiles. A typical element of X is $(x_1, x_2, \ldots, x_T)$ where x_i denotes health status in period i and T denotes the remaining number of years the individual decision maker will live until death. The x_i are elements of identical one-period sets of health outcomes A.

We assume the preference relation $\succeq$ over X to be a continuous weak order. A weak order is (i) complete: the individual decision maker can rank all health profiles, and (ii) transitive: if the individual decision maker prefers profile x to profile y ($x \succeq y$) and profile y to profile z ($y \succeq z$), then the individual should also prefer profile x to profile z ($x \succsim z$). Strict preference and indifference are denoted by $\succ$ and $\sim$ respectively.

Elements of X, the set of health profiles, are denoted by Roman characters x, y, etc. Elements of A, the one period sets of health outcomes, are denoted by Greek characters α, β etc. Constant alternatives are alternatives that give health outcome α in every period, and are denoted by α_c. We write $x_{-i}\alpha$ to denote the health profile x with x_i replaced by health outcome α. Similarly, $x_{-i,j}\alpha,\beta$ denotes health profile x with x_i replaced by α, and x_j replaced by β.

2.2. *Preference conditions*

Definition 2.1. The preference relation $\succsim$ is called *coordinate independent* (CI) if

$$(x_{-i}\alpha) \succsim (y_{-i}\alpha) \Leftrightarrow (x_{-i}\beta) \succsim (y_{-i}\beta) \text{ for all } x,y,i,\alpha,\beta.$$

The idea underlying CI is that if two alternatives have an identical health outcome in a certain period (have a coordinate in common), then the preference between these alternatives should be unaffected when that common health outcome is changed into another common health outcome. CI is also known by other names in the literature: e.g., independence (Debreu, 1960; Krantz et al., 1971), mutual preferential independence (Keeney and Raiffa, 1976).

Definition 2.2. The preference relation $\succsim$ is called *cardinally coordinate independent (CCI)* if for all $x,y,v,w,\alpha,\beta,\gamma,\delta,j$ and i, $(x_{-i}\alpha) \precsim (y_{-i}\beta)$ and $(x_{-i}\gamma) \succsim (y_{-i}\delta)$ and $(v_{-j}\alpha) \succsim (w_{-j}\beta)$ imply $(v_{-j}\gamma) \succsim (w_{-j}\delta)$.

The intuition behind this condition is as follows. Suppose α is preferred to β and γ is preferred to δ. One might say that in period i, the strength of preference of α over β is smaller than the strength of preference of γ over δ, since trading off β for α is not sufficient to compensate for getting x rather than y in all other time periods, whereas trading off δ for γ is sufficient. By CCI, if in period j the strength of preference of α over β is sufficient to compensate for getting v rather than w in all other time periods, then trading-off δ for γ is also sufficient. CCI establishes that trade-offs between health outcomes are not contradictory in different periods.

Definition 2.3. The preference order $\succsim$ is called *impatient* if

$$\alpha_c \succsim \beta_c \Rightarrow (x_{-i,i+1}\alpha,\beta) \succsim (x_{-i,i+1}\beta,\alpha) \text{ for all } x,\alpha,\beta.$$

According to definition 2.3, an individual is impatient if he prefers favourable outcomes to occur sooner rather than later. Impatience excludes the possibility that individuals prefer to postpone favourable outcomes because of the derivation of utility from the anticipation of future favourable outcomes.

Definition 2.4. The preference order $\succeq$ is called *stationary* if, for a constant alternative x, there exist health outcomes α and β such that for all time periods i:

$$(x_{-i}\beta) \sim (x_{-i+1}\alpha).$$

Stationarity has the effect of making the trade-off between health outcome β in time period i and health outcome α in time period $i+1$ invariant with respect to what time period i is. The trade-off between health outcomes occurring at different points in time depends only on the difference in time of occurrence between the health outcomes and not on the exact point in time at which they occur.

We are now ready to state a first theorem.

Theorem 2.1. The following two statements are equivalent:

(i) There exists a unique $0 < \pi \leq 1$, and a continuous function $V: A \to \mathbb{R}$, increasing up to positive affine transformations, such that the individual preference relation $\succeq$ over the set of health profiles X can be represented by

$$W(x) = \sum_{i=1}^{T} \pi^{i-1} V(x_i).$$

(ii) The preference relation $\succeq$ is a continuous weak order, it satisfies CCI, impatience and stationarity.

The proof of this theorem can be found in the appendix.

3. Intertemporal preferences under uncertainty

A widely held view in health economics is that, since risk is an essential element of health decision making, and no appropriate mechanisms exist for spreading the risk, individual attitudes towards risk should be incorporated in the decision making process both at the individual and group level (e.g., Ben-Zion and Gafni, 1983). A way to achieve this, following von Neumann and Morgenstern (1953), is to define preferences over lotteries over health outcomes rather than over the health outcomes themselves. We refer to lotteries over health outcomes as risky health outcomes.

3.1. Preliminaries

In the context of decision making under risk, the individual preference relation $\succeq_Z$ is defined over the set Z of simple probability distributions (lotteries) over X.

Elements of Z are denoted by capital Roman characters, P, Q, etc. Lotteries over A, the set of one period health outcomes, are denoted by P_i, Q_i, etc. P_i and Q_i are marginal probability distributions. We assume that the preference relation $\succeq_Z$ satisfies the von Neumann Morgenstern (vNM) axioms. These axioms are necessary and sufficient for the existence of a cardinal, real valued function $U: X \to \mathbb{R}$, the expectation of which represents $\succeq_Z$. It is important to realize that in vNM utility theory Z contains all degenerate probability distributions assigning probability one to an alternative. This induces a preference relation $\succeq$ over X. Note that U represents $\succeq$.

3.2. Preference conditions

In deriving the discounted utility representation, we make maximal use of the preference conditions defined in Section 2. An alternative approach would be to reformulate these conditions over risky health outcomes rather than over health outcomes (e.g. Fishburn, 1970 (section 11.4)). In our opinion defining preference conditions over the set of risky health outcomes makes the conditions less intuitive. We therefore restrict the use of conditions on the set of risky health outcomes to a minimum. However, one assumption on the set of risky health outcomes is necessary in order to relate risky health outcomes and health outcomes.

Definition 3.1. The preference relation $\succeq_Z$ on Z is called *additive independent* if

$$[P, Q \in Z, P_i = Q_i \text{ for } i = 1, \ldots, T] \Rightarrow P \sim_z Q.$$

Additive independence asserts that preferences over risky health outcomes depend only on the marginal probability of occurrence of each health outcome and not on their joint probability distribution. If two probability distributions result, at each point in time, in the same probability distribution over health outcomes, then by additive independence they should be indifferent.

Now a second theorem can be given.

Theorem 3.1. The following two statements are equivalent:

(i) There exists a unique $0 < \pi \leq 1$, and a continuous vNM utility function $U: A \to \mathbb{R}$, increasing up to positive affine transformations, such that the individual preference relation $\succeq$ over health profiles can be represented by

$$U(x) = \sum_{i=1}^{T} \pi^{i-1} U(x_i).$$

(ii) The preference relation $\succeq_Z$ over risky alternatives is a weak order, it satisfies vNM independence and Jensen continuity and additive independence.

Restricted to degenerate probability distributions, $\succeq$ satisfies CCI, impatience and stationarity.

The proof of this theorem can be found in the appendix.

4. A normative assessment of the preference conditions

Having identified the preference conditions underlying the discounted utility model, the question emerges of how appealing are these conditions. This section considers whether individuals *should* behave according to these preference conditions.

Coordinate independence is a strong assumption. It excludes complementarity of health states over times. Therefore phenomena such as coping and maximal endurable time (Sutherland et al., 1982), that depend on sequences of health states cannot be accounted for within the framework of the model. An example may clarify how CI excludes complementarity.

Suppose there are three points in time (three coordinates): $i = 1,2,3$ and three health states: good health (G), mediocre health (M) and poor health (P). Consider two choices: $A = (M_1, G_2, G_3)$ versus $B = (G_1, M_2, G_3)$ and $A' = (M_1, G_2, P_3)$ versus $B' = (G_1, M_2, P_3)$, where M_i stands for mediocre health in time period i. It is conceivable that an individual prefers A to B, because he would rather "get over" mediocre health quickly or because he is averse to changes in his health status. It is also conceivable that the same individual prefers B' to A', because he feels it is easier to cope with P_3 when his health decreases gradually over time. A preference for A over B and for B' over A' is caused by complementarity of health outcomes over time. Both variation aversion and coping relate to sequence effects. CI excludes the combination of A preferred to B and B' preferred to A'. The two choice situations differ only in the common third coordinate and, since by CI common coordinates cannot influence preference it follows that these two choice situations are equivalent.

When the coordinates i are states of the world rather than time points, CI is equivalent to Savage's (Savage, 1954) sure thing principle that preferences between alternatives should not be influenced by states of nature in which the two alternatives have common outcomes, regardless of what those common outcomes are. This is exactly what CI implies: common coordinates do not influence the preference relation.

The sure thing principle is theoretically less appealing when coordinates are points in time rather than states of nature. The traditional defence of the sure thing principle in the context of decision making under uncertainty (e.g. Samuelson, 1952), that something that never happens should not influence the value of something that actually does take place, does not carry over. In the points of time interpretation all time periods do occur.

It is a common belief in economics that individuals do indeed prefer benefits sooner rather than later, which supports impatience. Also, Olson and Bailey (1981) provide several normative arguments in defence of impatience. However, impatience excludes such effects as anticipation and dread. In the context of health decision making it does not seem irrational to prefer unpleasant events to happen sooner rather than later.

Stationarity lacks normative appeal as the time preference literature acknowledges. For example Fishburn and Rubinstein (1982) claim: "...we know of no persuasive argument for stationarity as a psychologically viable assumption" (p. 681). Similar views have been expressed by Koopmans (1960), Koopmans (1972). Stationarity requires the passage of time to have no influence on preferences. However, if an individual is indifferent between health improvement A now and health improvement B with a certain time delay x, why should this individual be indifferent between health improvement A in a year's time and health improvement B at time $x+1$ year?

Finally, additive independence is a strong condition. Additive independence excludes any complementarity of health outcomes in different time periods. For example, it requires that an individual is indifferent between two treatment scenarios A and B, where A results with probability 0.5 in "living 40 years in good health" and with probability 0.5 in "living 40 years in a poor health state P" and B results with probability 0.5 in "first living 20 years in good health followed by 20 years in P" and with probability 0.5 in "first living 20 years in P followed by 20 years in good health". In both treatment scenarios, in every year the individual has a probability of 0.5 of being in good health and a probability of 0.5 of living in health state P. Therefore, by additive independence, indifference should hold. However, some people could for example prefer treatment A because this gives the prospect of living the rest of their lives in good health, while others might prefer treatment B because this guarantees living 20 years in good health. For a more elaborate discussion of the appropriateness of additive independence in health decision making see Maas and Wakker (1994).

In summary, it appears that no persuasive arguments exist as to why an individual should behave according to the discounted utility model. It has been suggested by several authors (e.g. Weinstein, 1993) that the discounted utility model can be placed normatively on the same footing as the expected utility model. However, the translation of the expected utility model to the time context reduces the appeal of the underlying axioms and the discounted utility model also requires additional, restrictive, axioms.

5. A descriptive assessment of the preference conditions

This section considers the descriptive validity of the discounted utility model. First, an overview is given of the various factors that have been identified in empirical work as influencing individual intertemporal preferences. Second, direct empirical evidence is presented on the appropriateness of the discounted utility model in health decision making.

5.1. A decomposition of intertemporal preference

Olson and Bailey (1981), following Böhm–Bawerk, identify two ''influences'' which cause an individual to have a positive rate of time preference: decreasing marginal utility and pure time preference. Furthermore, they mention the influence of uncertainty on intertemporal preferences, but do not predict the sign of this effect. Gafni and Torrance (1984) have translated these effects to the case of a chronic health state. They identify the following three influences: (i) a quantity effect (decreasing marginal utility of health); (ii) a gambling effect, a consequence of the presence of uncertainty; and (iii) a pure time preference effect, reflecting the fact that individuals prefer to receive benefits sooner rather than later.

If present, all three of these effects will be properly handled by the discounted utility model. Decreasing marginal utility and the individual's attitude towards uncertainty are reflected by the shape of the utility function, and the pure time preference effect is incorporated in the discount factor. However, the analysis by Gafni and Torrance shows that separating these three different effects, which seems necessary in order to include them in a credible way in the discounted utility model, may prove to be a cumbersome task.

More recent empirical work [3] suggests that other influences besides the three mentioned above affect intertemporal preferences. Individuals generally prefer increasing profiles to decreasing profiles that are a permutation of these increasing profiles, both for wages (Frank and Hutchens, 1993; Loewenstein and Sicherman, 1991) and for other attributes (Loewenstein and Prelec, 1991), contrary to the predictions of the discounted utility model. Explanations of this fact distinguish sequences from single outcomes. Kahneman and Thaler (1991) identify adaptation and loss aversion as important influences on intertemporal preferences for sequences. Adaptation (also called anchoring) refers to the idea that the individual tends to consider the normal to be neutral, neither good nor bad. Adaptation is the foundation for Scitovsky's (Scitovsky, 1976) distinction between comforts, which become noticeable only when they are withdrawn, and pleasures, which are noticeable being distinct from the normal. Adaptation plays a central role in Loewenstein and Prelec's (1992) model of intertemporal choice. Streams of outcomes are evaluated as deviations from a reference vector rather than as being incorporated in existing plans. Loss aversion refers to the fact that the value function for losses is steeper than for gains. Adaptation in combination with loss

[3] See for example Loewenstein (1987), Loewenstein and Thaler (1989), Loewenstein and Prelec (1991, 1992, 1993) and Frank and Hutchens (1993).

aversion causes changes in the levels of well-being rather than absolute levels of well-being to be the real carriers of value for an individual.

Adaptation and loss-aversion are relevant for preferences over sequences. But condition CI, implied by the discounted utility model, excludes complementarity of health outcomes over time. Therefore, sequence effects cannot be incorporated in the model.

Loewenstein (1987) and Loewenstein and Prelec (1991) present empirical evidence on savouring and dread as factors influencing individual intertemporal preferences. Savouring refers to the utility experienced through the anticipation of future pleasures such as better health. Dread refers to the disutility experienced through the anticipation of future unattractive events such as poor health. Savouring and dread are excluded in the discounted utility model by the assumption that an individual always prefers to receive positive health benefits sooner rather than later (impatience).

Adaptation, loss-aversion, savouring and dread challenge the discounted utility model also in another way. Their existence makes the isolation of the pure time preference effect very complicated. Analyses based on the discounted utility model need to isolate the pure time preference effect in order to determine the appropriate discount rate. However, it seems impossible to disentangle the separate influences of quantity, uncertainty, pure time preference, adaptation, loss-aversion, savouring and dread on individual intertemporal preferences. The discussion about the pure rate of time preference resembles the discussion about the concept of intrinsic risk attitude (Schoemaker, 1993): the concept is interesting as a theoretical construct, but unobservable in reality.

The confounding of various influences on intertemporal preferences can explain the anomalous preference patterns that have been observed with respect to health. For example, Redelmeier and Heller (1993) observe that a large proportion of their study population effectively applies a negative discount rate but that does not necessarily imply a negative pure rate of time preference. It can be explained by other influences, because the design used by Redelmeier and Heller (1993) is not capable of isolating the pure rate of time preference. Perhaps because health is a good with a time dimension inextricably bound to it, no experimental study can change the timing of the event without changing other factors, so attempts to measure the pure rate of time preference are likely to prove futile.

5.2. Direct evidence

Studies that have investigated the predictions of the discounted utility model with respect to health decision making have typically rejected the model. Lipscomb (1989) studied preferences over health streams by means of both the discounted utility model and a more general strategy (i.e., imposing less restrictions on the individual preference relation) which he refers to as the scenario strategy. Lipscomb observed some conflicting predictions, in the sense that the scenario strategy predicted a preference for health profile *A* over *B* where the discounted utility model predicted a preference for *B* over *A*. Since Lipscomb's scenario strategy imposes fewer restrictions, it will in general better predict choices, and in the case of conflicting predictions the discounted utility model seems to lead to the wrong prediction.

The results of recent empirical studies, attempting to elicit the rate of discount individuals apply to health outcomes, cast further doubts on the validity of the discounted utility model in modelling individual intertemporal preferences for health outcomes. The studies by Redelmeier and Heller (1993), Olsen (1993a), Mackeigan et al. (1993) and Cairns (1994) all reject the constant rate discounted utility model for the time preferences of such diverse groups as students, physicians, health policy makers and members of the general public. The pattern that emerges from these studies is a high discount rate for more proximate years and a lower discount rate for more distant years.

6. Variable rate discounted utility models

Given the deficiencies of the constant rate discounted utility model, Olsen (1993a) and Harvey (1994) among others, have suggested replacing the constant rate discounted utility (CRDU) model by a variable rate discounted utility (VRDU) model. The axiomatization of the VRDU model follows readily from the analyses of Sections 2 and 3. In the context of Section 2, condition CCI in addition to the structural assumptions is sufficient to obtain a general VRDU model. In the context of Section 3, additive independence has to be imposed as well. VRDU models are more general than the constant rate discounted utility model. Stationarity is no longer imposed, and therefore intertemporal trade-offs no longer need to be invariant with respect to the passage of time. Impatience does not need to be imposed, unless the discount function is to be a decreasing function of time.

Because VRDU models make fewer assumptions with respect to individual preferences, they are better able to predict observable data. However, such required conditions as additive independence are still strong as has been argued in Sections 4 and 5. Since CCI implies CI, sequence effects are still excluded. Finally, like its constant rate counterpart, the variable rate discounted utility model needs information on the pure rate of time preference, information that may be difficult to retrieve.

An individual whose preferences satisfy a VRDU model at any point in time faces another problem: varying discount rates may lead to dynamically inconsistent preferences (Strotz, 1956; Hammond, 1976). Suppose an individual must choose between two scenarios both involving three periods. Scenario A yields the sequence of health benefits (0.8, 0.6, 0.4), and scenario B yields the sequence (0.8, 0.4, 0.61). Suppose that the individual is a variable rate discounted utility maximizer at any time period. Assume that the discount rate for the first period is

0%, that the discount rate for the second period is 10% and for the third period 4%. It is easily checked that a VRDU maximizer will prefer scenario B.

Suppose the individual reconsiders his choice after the first period. Suppose further that the individual can switch programs at a certain cost. Since benefits in the first period are equal in the two scenarios, the individual can concentrate on the future benefits of the two programs. Recalculating his discounted utility, the individual does not discount the benefits occurring in (what was) the second period and applies the discount rate of 10% to the benefits occurring in (what was) the third period. The individual will now prefer scenario A and will pay any amount up to the sum of money which is equivalent to the utility difference between the two programs to be able to switch. Similar examples involving more than three periods can be constructed, in which the individual will pay an amount of money every period to be able to switch scenarios, only to end up in the scenario he already preferred in the first period, but not after having lost a good deal of money.

7. Concluding remarks

We have argued that the discounted utility model is inappropriate in modelling individual intertemporal preferences, both for certain health outcomes and for uncertain health outcomes. First, the axiom system of the discounted utility model is restrictive. Second, information about the individual pure rates of time preference for health, which is necessary for the discounted utility model, is unlikely to be retrievable. There is an additional problem in discounting health outcomes: the possibility of double discounting (Krahn and Gafni, 1993; Gafni, 1994). Since health outcomes cannot be defined without reference to time duration, and utility assessment procedures typically introduce a time dimension (Torrance, 1986; Torrance and Feeny, 1989), individuals may incorporate their time preferences at least to some extent in the assessment of the utility of various health outcomes. Fully discounting health outcomes in such a situation would not be appropriate.

Where do these negative conclusions lead? One possibility is to relax the preference conditions underlying the discounted utility model to take individual intertemporal preferences for health outcomes into account in a more realistic way in health care program evaluation. On the other hand, relaxing preference conditions necessarily implies assessing more parameters. At every stage, the trade off between theoretical soundness and practical feasibility has to be made.

The VRDU model does relax the preference conditions of the CRDU model. However, the VRDU model still does not allow complementarity between time periods, which is possibly the most restrictive assumption of the CRDU model. Complementarity between times can be introduced in the model by following one of two approaches. One possibility is to extend the utility function by incorporating factors like habit formation (Pollak, 1970; Constantinides, 1990), the rate of benefit change (Frank and Hutchens, 1993) or preference/aversion for utility variation between adjacent periods (Gilboa, 1989). Gilboa's model is an attempt to apply the Choquet expected utility models, which have been successful in decision making under uncertainty, to the time context. In the model where preferences concern risky health outcomes, preferences for health outcomes can be made to depend on the joint probability distribution, albeit in a limited sense, by relaxing additive independence to mutual utility independence. Miyamoto and Eraker (1988) have found that utility independence generally holds in the health context.

Alternatively, the utility of health scenarios can be assessed directly by evaluating the whole stream of health outcomes. By not evaluating health outcomes separately, this approach in fact rejects coordinate independence. This is the idea behind Lipscomb's scenario strategy as well as the HYE (Gafni, 1995). A disadvantage of scenario-based measures is that their "refusal" to evaluate health outcomes separately excludes the evaluation of health scenarios by short cuts. Whereas approaches based on coordinate independence need only assess a limited number of health states, in a scenario strategy every scenario must be assessed separately. This might limit their applicability in complex medical decision problems involving many possible health outcomes.

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Appendix A. Proof of theorem 2.1

A.1. Mathematical structure

The set of alternatives, X, is assumed to be a Cartesian product of the identical one period sets A: $X = A^T$. Time periods i are elements of a finite index set $I = \{1,\ldots,T\}$ with $T \in \mathbb{N}$. The following structural assumptions are made with respect to A and X: (i) A is a connected and separable topological space; [4] (i) X is endowed with the product topology. Connectedness ensures that every continuous function from X to $\mathbb{R}$ has an interval as its image, so that this image has no

[4] In fact topological separability does not have to be assumed if more than one time period is essential (Krantz et al., 1971 (section 6.11.1); Vind, 1990; Wakker, 1989 (Remark III.7.1)).

holes. The weak order $\succeq$ defined on X is taken as primitive. A weak order $\succeq$ is complete ($x \succeq y$ or $y \succeq x$ for all $x, y \in X$) and transitive (if $x \succeq y$ and $y \succeq z$ then $x \succeq z$). This implies that the indifference relation $\sim$, defined as both $x \succeq y$ and $y \succeq x$, is an equivalence, i.e. it is symmetric ($x \sim y \Leftrightarrow y \sim x$), reflexive ($x \sim x$) and transitive). Strict preference $x \succ y$ is defined as $x \succeq y$ and not $y \succeq x$. We assume that $\succeq$ is continuous: $\{x : x \succeq y\}$ and $\{x : x \preceq y)$ are closed for all $y \epsilon X$. Continuity of the preference relation ensures that, if a function W exists that represents the preference relation, i.e. $W: X \to \mathbb{R}$ satisfies $x \succeq y \Leftrightarrow W(x) \geq W(y)$, then this function makes no jumps. The topological assumptions and the assumption that $\succeq$ is a continuous weak order are necessary and sufficient for the existence of a continuous representing function $W: X \to \mathbb{R}$ (Debreu, 1954).

We assume that there are at least two time periods and that every time period is essential, i.e. $x_{-i}\alpha \succ x_{-i}\beta$ for some health outcomes α and $\beta \in A$ and for all i.

A.2. Proof of theorem 2.1

That (i) implies (ii) is straightforward. Hence we assume that (ii) holds and derive (i).

Definition A1.1. The preference order $\succeq$ is called *persistent* if

$$(x_{-i}\alpha) \succeq (x_{-i}\beta) \Leftrightarrow (y_{-j}\alpha) \succeq (y_{-j}\beta) \text{ for all } x, y, \alpha, \beta, i, j.$$

Persistence of the preference order asserts that preferences for health outcomes are identical in every time period. Persistence excludes to some extent a preference for variety. This can be seen for example by setting all elements of y in the above definition equal to α.

Persistence is implied by CCI. Set $x = y$, $v = w$, $\alpha = \beta$. By reflexivity of $\sim$: $x_{-i}\alpha \sim x_{-i}\alpha$ and $v_{-j}\alpha \sim v_{-j}\alpha$. So $x_{-i}\alpha \preceq x_{-i}\alpha$ and $v_{-j}\alpha \succeq v_{-j}\alpha$ both hold. Now $v_{-j}\gamma \succeq v_{-j}\delta$ follows from CCI.

By the structural assumptions being made, by CCI and by lemma IV.2.5 in Wakker (1989), we know from theorem IV.2.7 in Wakker (1989) that the preference relation can be represented by $x \succeq y \Leftrightarrow \Sigma\lambda_i V(x_i) \geq \Sigma\lambda_i V(x_i)$ with the λ_i uniquely determined and V continuous and unique up to positive linear transformations.

As shown above, CCI $\Rightarrow$ persistence. By persistence we cannot have $x_{-i}\alpha \succeq x_{-i}\beta$ and $x_{-j}\alpha \prec x_{-j}\beta$. Thus we cannot have

$$\sum_{k \neq i} \lambda_k V(x_k) + \lambda_i V(\alpha) \geq \sum_{k \neq i} \lambda_k V(x_k) + \lambda_i V(\beta)$$

and

$$\sum_{l \neq j} \lambda_l V(x_l) + \lambda_j V(\alpha) < \sum_{l \neq j} \lambda_l V(x_l) + \lambda_j V(\beta).$$

From this it follows that either all λ_j's are positive or all λ_j's are negative. If all λ's are negative, replace V by $-V$ and λ_j by $-\lambda_j$. So all λ_j's are positive. Then it automatically follows that if $\alpha_c \succeq \beta_c$ then $V(\alpha) \geq V(\beta)$. By impatience if $\alpha_c \succeq \beta_c$ then $x_{-i,i+1}\alpha,\beta \succeq x_{-i,i+1}\beta,\alpha$. So

$$\sum_{k\neq i,i+1} \lambda_k V(x_k) + \lambda_i V(\alpha) + \lambda_{i+1} V(\beta) \geq \sum_{k\neq i,i+1} \lambda_k V(x_k) + \lambda_i V(\beta) + \lambda_{i+1} V(\alpha)$$

$$\Rightarrow \lambda_i(V(\alpha) - V(\beta)) \geq \lambda_{i+1}(V(\alpha) - V(\beta)).$$

Thus $\lambda_i \geq \lambda_{i+1}$. Set $\lambda_1 = 1$.
Now by stationarity

$$(z_{-i}\beta) \sim (z_{-i+1}\alpha) \text{ for all } i,i+1.$$

The existence of such a z follows from restricted solvability, which by lemma III.3.3 in Wakker (1989) is implied by the topological assumptions and by $\succeq$ being a continuous weak order. Restricted solvability asserts that for every $x_{-i}\alpha \succeq y \succeq x_{-i}\gamma$ there exists β such that $x_{-i}\beta \sim y$. Take $y = z_{-i+1}\alpha$, $z = x$ and select some γ_c such that $\alpha_c \succeq \beta_c \succeq \gamma_c$.

Because V is unique up to positive linear transformations, we can set $V(Z)$ equal to zero. Following from the assumption that all time periods are essential, there exist α and β such that $V(\alpha)$, $V(\beta) > 0$. Now from stationarity $(z_{-1}\beta) \sim (z_{-2}\alpha) \Rightarrow V(\beta) = \lambda_2 V(\alpha) \Rightarrow \lambda_2 = V(\beta)/V(\alpha)$. Apply stationarity again to get $(z_{-2}\beta) \sim (z_{-3}\alpha) \Rightarrow \lambda_2 V(\beta) = \lambda_3 V(\alpha) \Rightarrow \lambda_3 = \lambda_2[V(\beta)/V(\alpha)] \Rightarrow \lambda_3 = [V(\beta)/V(\alpha)]^2$. Set $\pi = [V(\beta)/V(\alpha)]$. Then the constant discount rate model follows. Since by impatience $\pi \leq \lambda_1 = 1$ and every $\lambda_j > 0$ as established above, $0 < \pi \leq 1$.

Appendix B. Proof of theorem 3.1

B.1. Mathematical structure

Z is defined as the set of all simple probability measures on X. A simple probability measure on X is a real-valued function P defined on the set of all subsets of X such that: (i) $P(B) \geq 0$ for every $B \subseteq X$; (ii) $P(X) = 1$; (iii) $P(B \cup C) = P(B) + P(C)$ when $B,C \subseteq X$ and $B \cap C = \emptyset$; (iv) $P(B) = 1$ for some *finite* $B \subseteq X$. A typical element of Z is denoted by $(p^1,x^1;\ldots;p^m,x^m)$ where, for each j, alternative x^j results with probability p^j and m can be any natural number. Elements of Z are denoted by capital Roman characters P,Q etc. Since Z contains many simple probability distributions, risky health outcomes can be mixed, or more formal, Z is closed under convex combinations: if $P,Q \in Z$ and $\lambda \in [0,1]$ then $\lambda P + (1-\lambda)\ Q \in Z$, where $\lambda P + (1-\lambda)Q$ is the lottery $(\lambda p^1 + (1-\lambda)q^1,x^1;\ldots;\lambda p^m + (1-\lambda)q^m,x^m)$.

The preference relation $\succeq_Z$ is defined on Z. $\succeq_z$ is assumed to be a weak order. Furthermore, we impose the following two axioms (Jensen, 1967):

1. vNM independence: $(P \succeq_z Q,\ 0 < \mu < 1) \Leftrightarrow (\mu P + (1-\mu)R) \succeq_z (\mu Q + (1-\mu)R)$ for all $P,Q,R \in Z$
2. Jensen continuity: $(P \succ Q,\ Q \succ R) \Rightarrow \mu P + (1-\mu)R \succ_z Q$ and $Q \succ_z \rho P + (1-\rho)R$ for some $\mu, \rho \in (0,1)$.

vNM independence is widely regarded to be the core of expected utility theory. It says that if P is weakly preferred to Q then any convex combination of P and R should be weakly preferred to a similar convex combination of Q and R. Jensen continuity is an Archimedean condition, which asserts that, for $P \succ_z Q \succ_z R$, there are values μ and ρ such that the convex combination of P and R is preferred to Q respectively Q is preferred to the convex combination of P and R.

Define Y as the set of simple probability distributions on the one period sets of health outcomes A. A marginal probability measure P_i is defined on A as: if $B \subset A$, then $P_i(B) = P(X: x_i \in B)$. A preference relation $\succeq$ is defined on Y from $\succeq$ on Z in the following way:

$$R \succeq_z S \Leftrightarrow P \succeq_z Q \text{ for } P,Q \in Z \text{ and } R,S \in Y, \text{ such that}$$

$P_i = R$ and $Q_i = S$ for all time points i and P assigns probability one to a constant x.

B.2. Proof of theorem 3.1

(i) $\Rightarrow$ (ii) is again straightforward. Hence, assume (ii). To derive is (i).

Because both $W = \Sigma \pi^{i-1} V(x_i)$ and U represent a preference relation $\succeq$ over X, they are related by a strictly increasing transformation. Under the assumptions of Section 2, W is a continuous additive representation of $\succeq$ over X. Now, if U can also be written as a continuous additive representation of $\succeq$, then, by cardinality of V and U, U is a linear transform of V and can be taken identical to V. By additive independence $U(x) = \Sigma_i U_i(x_i)$ (this result has been proved by Fishburn (1965)). By theorem 3.2 in Maas and Wakker (1994), U is continuous. Thus, U can be set equal to W: $U(x) = \Sigma_i V_i(x_i)$. Then apply the proof of theorem 2.1. This gives the desired result.

References

Ben-Zion, U. and A. Gafni, 1983, Evaluation of public investment in health care: Is the risk irrelevant?, Journal of Health Economics 2, 161–165.

Cairns, J., 1994, Valuing future benefits, Health Economics 3, 221–229.

Constantinides, G.M., 1990, Habit formation: A resolution of the equity premium puzzle, Journal of Political Economy 98, 519–543.

Debreu, G., 1954, Representation of preference ordering by a numerical function, in: R.M Thrall, C.H. Coombs and R.L. Davids, eds., Decision processes (Wiley, New York), 159–165.

Debreu, G., 1960, Topological methods in cardinal utility theory, Ch. 2 in: K.J. Arrow, S. Karlin and P. Suppes, eds., Mathematical methods in the social sciences (Stanford University Press) 16–26.

Fishburn, P.C., 1965, Independence in utility theory with whole product sets, Operations Research 13, 28–45.

Fishburn, P.C., 1970, Utility theory for decision making (Wiley, New York).

Fishburn, P.C. and A. Rubinstein, 1982, Time preference, International Economic Review 23, 677–694.

Frank, R. and R. Hutchens, 1993, Wages, seniority, and the demand for rising consumption profiles, Journal of Economic Behavior and Organization 21, 251–276.

Gafni, A., 1994, The standard gamble method: What is being measured and how it is interpreted. Health Services Research 29, 207–224.

Gafni, A., 1995, Time in health: Can we measure individual's pure time preference?, Medical Decision Making 15, 31–37.

Gafni, A. and G.W. Torrance, 1984, Risk attitude and time preference in health, Management Science 30, 440–451.

Gilboa, I., 1989, Expectation and variation in multi-period decisions, Econometrica 57, 1153–1169.

Hammond, P., 1976, Changing tastes and coherent dynamic choice, Review of Economic Studies 43, 159–173.

Harvey, C.M., 1994, The reasonableness of non-constant discounting, Journal of Public Economics 53, 31–51.

Jensen, N.E., 1967, An introduction to Bernoullian utility theory: I. Utility functions, Swedish Journal of Economics 69, 163–183.

Kahneman, D. and R. Thaler, 1991, Economic analysis and the psychology of utility: Applications to compensation policy, American Economic Review Proceedings 81, 341–346.

Keeney, R.L. and H. Raiffa, 1976, Decisions with multiple objectives: Preferences and value tradeoffs (Harvard University Press, Cambridge, MA).

Koopmans, T.C., 1960, Stationary ordinal utility and impatience, Econometrica 28, 287–309.

Koopmans, T.C., 1972, Representation of preference orderings over time, Ch. 4 in: C.B. McGuire and R. Radner, eds., Decision and organization: A volume in honor of Jacob Marschak (North Holland, Amsterdam) 79–100.

Koopmans, T.C., P.A. Diamond and R.E. Williamson, 1964, Stationary utility and time perspective, Econometrica 32, 82–100.

Krahn, M. and A. Gafni, 1993, Discounting in the economic evaluation of health care interventions, Medical Care 31, 403–418.

Krantz, D.H., R.D. Luce, P. Suppes and A. Tversky, 1971, Foundations of measurement, Vol. 1: Additive and polynomial representations (Academic Press, New York).

Lind, R.C., 1990, Reassessing the government's discount rate policy in the light of new theory and data in a world economy with a high degree of capital mobility, Journal of Environmental Economics and Management 18, S8–28.

Lipscomb, J., 1989, Time preference for health in cost-effectiveness analysis, Medical Care 27, S233–S253.

Loewenstein, G., 1987, Anticipation and the value of delayed consumption, Economic Journal 97, 666–684.

Loewenstein, G., 1988, Frames of mind in intertemporal choice, Management Science 34, 200–214.

Loewenstein, G. and D. Prelec, 1991, Negative time preference, American Economic Review Proceedings 81, 347–352.

Loewenstein, G. and D. Prelec, 1992, Anomalies in intertemporal choice: Evidence and an interpretation, Quarterly Journal of Economics 107, 573–597.

Loewenstein, G. and D. Prelec, 1993, Preferences for sequences of outcomes, Psychological Review 100, 91–108.

Loewenstein, G. and N. Sicherman, 1991, Do workers prefer increasing wage profiles?, Journal of Labor Economics.

Loewenstein, G. and R. Thaler, 1989, Anomalies: Intertemporal choice, Journal of Economic Perspectives 3, 181–193.

Maas, A. and P.P. Wakker, 1994, Additive conjoint measurement for multiattribute utility, Journal of Mathematical Psychology 38, 86–101.

Mackeigan, L.D., L.N. Lawson, J.R. Drangalis, J.L. Bootman and L.R. Burns, 1993, Time preference for health gains versus health losses, Pharmaco Economics 3, 374–386.

Miyamoto, J.M. and S.A. Eraker, 1988, A multiplicative model of the utility of survival duration and health quality, Journal of Experimental Psychology: General 117, 3–20.

Olsen, J.A., 1993a, Time preference for health gains: An empirical investigation, Health Economics 2, 257–265.

Olsen, J.A., 1993b, On what basis should health be discounted, Journal of Health Economics 12, 39–53.

Olson, M. and M.J. Bailey, 1981, Positive time preference, Journal of Political Economy 89, 1–25.

Pollak, R.A., 1970, Habit formation and dynamic demand functions, Journal of Political Economy 78, 745–763.

Redelmeier, D.A. and D.N. Heller, 1993, Time preference in medical decision making and cost effectiveness analysis, Medical Decision Making 13, 212–217.

Savage, L.J., 1954, The foundations of statistics (Wiley, New York).

Samuelson, P.A., 1952, Probability, utility and the independence axiom, Econometrica 20, 670–678.

Schoemaker, P.J.H., 1993, Determinants of risk-taking: Behavioral and economic views, Journal of Risk and Uncertainty 6, 49–73.

Scitovsky, T., 1976, The joyless economy (Oxford University Press).

Strotz, R.H., 1956, Myopia and inconsistency in dynamic utility maximization, Review of Economic Studies 23, 165–180.

Sutherland, H.J., H. Llewellyn-Thomas, N.F. Boyd and J.E. Till, 1982, Attitudes toward quality of survival: The concept of "maximal endurable time", Medical Decision Making 2, 299–309.

Thaler, R., 1981, Some empirical evidence on dynamic consistency, Economic Letters 8, 201–207.

Torrance, G.W., 1986, Measurement of health state utilities for economic appraisal, Journal of Health Economics 5, 1–30.

Torrance, G.W. and D. Feeny, 1989, Utilities and quality adjusted life-years, International Journal of Technology Assessment in Health Care 8, 559–578.

Vind, K., 1990, Additive utility functions and other special functions in economic theory, Discussion paper 90–21, Institute of Economics, University of Copenhagen.

von Neumann, J. and O. Morgenstern, 1953, Theory of games and economic behaviour, 3rd edn. (Wiley, New York).

Wakker, P.P., 1989, Additive representations of preferences: A new foundation of decision analysis (Kluwer, Dordrecht).

Weinstein, M.C., 1993, Time-preference studies in the health care context, Medical Decision Making, 218–219.