Diagnostic and Therapeutic ambiguity under maxmin expected utility, α-maxmin expected utility, and contraction expected utility

This note will show that the results derived in Berger, Bleichrodt, and Eeckhoudt (2013), BBE from now on, also hold under three alternative ambiguity models: maxmin expected utility, α-maxmin expected utility, and contraction expected utility. Notation is as in BBE.

In the influential maxmin expected utility model of Gilboa and Schmeidler (1989) the decision maker maximizes the minimum of the expected utilities for the probabilities that he considers possible. Maxmin expected utility is a special case of the smooth model with $\varphi(x) = (1 - \exp(-\alpha x))/\alpha$ and $\alpha \to \infty$ and, hence, our conclusions also hold under this model. Maxmin EU captures an extreme form of ambiguity aversion in the sense that the decision maker bases his decisions on the worst case scenarios. In the model of diagnostic ambiguity, for example, this means that he bases his decisions on the highest probability of illness (p_2 in the model of Section 2 of BBE and p_n in the model of Appendix A of BBE) and hence, will be strongly inclined to choose treatment.

A less pessimistic model is α-maxmin EU (Ghirardato et al., 2004; Jaffray, 1989, Eeckhoudt and Jeleva, 2004) in which preferences are represented by a linear combination of the minimum and the maximum of the expected utilities. In the model of diagnostic ambiguity, expected utility declines with the probability of illness and, hence, the maximum expected utility is obtained for p_1, the lowest probability of illness in Δ, and the minimum expected utility for p_n, the highest probability of illness in Δ. Hence, under α-maxmin:

$$V^T = \alpha(p_n U(H_s^T) + (1 - p_n)U(H_h^T)) + (1 - \alpha)(p_1 U(H_s^T) + (1 - p_n)U(H_h^T)) \quad (1)$$

$$V^{NT} = \alpha\big(p_n U(H_s^{NT}) + (1 - p_n)U(H_h^{NT})\big) + (1 - \alpha)\big(p_1 U(H_s^{NT}) + (1 - p_n)U(H_h^{NT})\big) \quad (2)$$

The parameter α reflects the degree of ambiguity aversion. For a given set of beliefs, the higher is α, the more ambiguity averse is the decision maker. From Eqs. (1) and (2) it immediately follows that we can write $V^T = qU(H_s^T) + (1 - q)U(H_h^T)$ and $V^{NT} = qU(H_s^{NT}) + (1 - q)U(H_h^{NT})$ with $q = \alpha p_n + (1 - \alpha)p_1$. q can be interpreted as the probability of illness used by the decision maker in evaluating treatment versus no treatment. From Figure 1 in BBE, it is immediately obvious than an increase in ambiguity aversion (an increase in α) corresponds with an increase in q and, hence, with an increase in the attractiveness of treatment. Consequently, the conclusion that an increase in diagnostic ambiguity aversion generates an increase in the propensity to treat is also true under α-maxmin.

Regarding therapeutic ambiguity, under α-maxmin the decision maker compares

$$V^T = \alpha\big(p_n U(H_s^{T-}) + (1 - p_n)U(H_s^{T+})\big) + (1 - \alpha)\big(p_1 U(H_s^{T-}) + (1 - p_1)U(H_s^{T+})\big) \quad (3)$$

with $V^{NT} = U(H_s^{NT})$). Similar to the case of diagnostic ambiguity, $q = \alpha p_n + (1 - \alpha)p_1$ can be interpreted as the failure rate used by the decision maker to evaluate the two options. An increase in ambiguity aversion leads to an increase in this subjective probability and we notice from Figure 6 in BBE that this makes the option no treatment more attractive.

A drawback of the α-maxmin model is that it does not contain expected utility as a special case (unless the set of beliefs is a singleton and there is no ambiguity). Gajdos et al. (2008) suggested a model that generalizes maxmin EU and that does contain expected utility as a special case. In their model the benfits of treatment and no treatment are:

$$V^T = \alpha(p_n U(H_s^T) + (1 - p_n)U(H_h^T)) + (1 - \alpha)(E(\tilde{p})U(H_s^T) + (1 - E(\tilde{p}))U(H_h^T)) \quad (8)$$

$$V^{NT} = \alpha(p_n U(H_s^{NT}) + (1 - p_n)U(H_h^{NT})) + (1 - \alpha)(E(\tilde{p})U(H_s^{NT}) + (1 - E(\tilde{p}))U(H_h^{NT})) \qquad (9)$$

where $E(\tilde{p})$ denotes the expected value of the probabilities of illness that the decision maker considers possible (i.e. the expectation of the p in Δ). The model of Gajdos et al. (2008) is a linear combination of the minimum expected utility with respect to the probabilities p in Δ and the expected utility (the case of ambiguity neutrality). As in α-maxmin, the parameter α captures the decision maker's ambiguity aversion, with larger values of α corresponding with more ambiguity aversion. We can write $V^T = qU(H_s^T) + (1 - q)U(H_h^T)$ and $V^{NT} = qU(H_s^{NT}) + (1 - q)U(H_h^{NT})$ with $q = \alpha p_n + (1 - \alpha)\, E(\tilde{p})$. An increases in ambiguity aversion corresponds with an increase in q and, hence, with an increase in the attractiveness of treatment.

The analysis of therapeutic ambiguity is similar as under α-maxmin but with the perceived failure rate $q = \alpha p_n + (1 - \alpha)E(\tilde{p})$.

References

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