Resolving Inconsistencies in Utility Measurement Under Risk: Tests of Generalizations of Expected Utility

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This paper explores inconsistencies that occur in utility measurement under risk when expected utility theory is assumed and the contribution that prospect theory and some other generalizations of expected utility can make to the resolution of these inconsistencies. We used five methods to measure utilities under risk and found clear violations of expected utility. Of the theories studied, prospect theory was the most consistent with our data. The main improvement of prospect theory over expected utility was in comparisons between a riskless and a risky prospect (riskless-risk methods). We observed no improvement over expected utility in comparisons between two risky prospects (risk-risk methods). An explanation for the latter observation may be that there was less distortion in probability weighting in the interval $[0.10, 0.20]$ than has commonly been observed.

1. Introduction

Utility measurement is an important tool for decision analysis and helps clients to make better-informed choices. To obtain valid and consistent utility measurements, the decision analyst needs a theory that is descriptively valid so that he can then use these utilities as inputs into a model, e.g., a decision tree, that will help the client to make a choice truly representing his preferences.

This paper explores inconsistencies in the measurement of utilities under risk and tries to find a theory of preferences that can solve these inconsistencies. Risky utilities are widely used in decision analysis. The main examples are the *probability equivalence* (PE) method, in which people are asked to state a probability in a risky prospect that makes them indifferent between this risky prospect and a given sure outcome, and the *certainty equivalence* (CE) method, in which people are asked to state a certain outcome that makes them indifferent between this outcome and a given risky prospect. The appealing feature of using methods involving risk is that they allow the analyst to

incorporate people's attitudes toward risk into decision analysis.

When measuring utility, investigators typically assume that responses conform to expected utility theory. A rationale for adopting expected utility is that decision analysis is essentially a prescriptive exercise and that expected utility is the dominant prescriptive theory of decision under risk. The problem with this point of view is that in most practical applications measuring utilities is a descriptive task, and the descriptive deficiencies of expected utility are widely documented (Starmer 2000). Using expected utility to analyze responses to utility measurement tasks in spite of its poor descriptive standing can lead to biased utilities, and decision analyses based on these biased utilities may result in incorrect recommendations.

Several studies have shown that utility measurement based on expected utility leads to inconsistencies both across and within methods. Hershey and Schoemaker (1985) showed that PE measurements result in systematically higher utilities than CE measurements. McCord and de Neufville (1986) found

that the utility function elicited by the CE method depends on the value at which probability is fixed. In the health domain, several authors have shown that assessment procedures that are theoretically equivalent under expected utility produce systematically different utilities (Llewellyn-Thomas et al. 1982, Rutten-van Mölken et al. 1995, Bleichrodt 2001, Oliver 2003, Pinto-Prades and Abellan-Perpiñan 2005). For example, Pinto-Prades and Abellan-Perpiñan (2005) found that under expected utility, the utility of a given health state varied between 0.48 and 0.80 depending on the assessment procedure that was used. Such large discrepancies can have significant effects for practical decision analyses.

One solution for the observed inconsistencies was suggested by Bleichrodt et al. (2001). They argued that people's preferences are affected by biases. The biases they considered were (1) probability weighting, the nonlinear evaluation of probabilities, and (2) loss aversion, the finding that people are more sensitive to losses than to gains of the same size. Both biases have been modelled by prospect theory (Kahneman and Tversky 1979, Tversky and Kahneman 1992), currently the main descriptive theory of decision under risk. Bleichrodt et al. (2001) proposed new formulas based on prospect theory to evaluate answers to PE and CE measurements and showed that these new formulas were able to resolve the systematic discrepancy between PE and CE utilities.

PE and CE are examples of methods in which a riskless prospect is compared with a risky prospect (*riskless-risk methods*). Several empirical studies have observed that violations of expected utility primarily occur when one of the prospects under consideration is riskless; expected utility's descriptive record is much better when both prospects are risky (Conlisk 1989, Camerer 1992, Harless and Camerer 1994, Wu and Gonzalez 1996, Starmer 2000). Hence, it is of interest to examine the performance of prospect theory when we include methods that compare two risky prospects (*risk-risk methods*). It would be important for practical decision analysis in case we observed that expected utility could explain the data from risk-risk measurements. Then, no information on probability weighting and loss aversion is required and preferences can be measured under expected utility by using risk-risk methods such as the lottery equivalence method (McCord and de Neufville 1986).

In this paper, we compare five elicitation methods: three riskless-risk methods and two risk-risk methods. The aims of our study were threefold: first, to replicate the inconsistency between PE and CE under expected utility, and to compare PE and CE with a third riskless-risk method; second, to test whether expected utility leads to consistent utilities for the two risk-risk methods; and third, to explore whether

prospect theory could explain any inconsistencies across the five methods that were observed under expected utility. Our study was motivated by the idea that if the five assessment procedures yield consistent results when analyzed under a particular preference theory, but inconsistent results under another, then the theory under which consistency is found is supported as a descriptive theory.

We studied the performance of expected utility and prospect theory in the health domain. The main reason we focused on health is practical relevance. Health is an important area of applied decision analysis (the majority of applied decision analyses have been in the health domain; see Keller and Kleinmuntz 1998, Smith and von Winterfeldt 2004) and risky methods are widely used in medical decision analysis.

Under expected utility, we found that the obtained utilities varied across the three riskless-risk methods and were larger than the obtained utilities under the risk-risk methods. Prospect theory fitted the data better than expected utility. The main improvement of prospect theory over expected utility was in the evaluation of the riskless-risk methods. For risk-risk methods, we found no violations either for expected utility or for prospect theory and, hence, prospect theory did not improve over expected utility. Because neither expected utility nor prospect theory could entirely explain the data, we also considered several other generalizations of expected utility: rank-dependent utility (Quiggin 1981), disappointment aversion (Gul 1991), and two recently proposed gambling effect models (Bleichrodt and Schmidt 2002, Diecidue et al. 2004). None of these nonexpected utility models fitted the data as well as prospect theory.

In what follows, §2 introduces notation and briefly explains prospect theory. Section 3 describes the five elicitation methods used. Section 4 analyzes the predictions from expected utility and prospect theory and shows how utilities were computed under these two theories. Section 5 describes the experiment that we performed and §6 its results. Section 7 discusses the main findings and limitations of our study. Section 8 concludes the paper. The appendix contains derivations of results introduced in the text.

2. Notation and Outline of the Models

We study preferences over chronic health states. Chronic health states will be written as (Q, T), denoting T years in health state Q. Because the experiment reported later only invokes prospects with at most two different chronic health states, we will restrict the analysis to such *binary prospects*. Let $(p: (Q_1, T_1); (Q_2, T_2))$ denote the prospect that gives

(Q_1, T_1) with probability p and (Q_2, T_2) with probability $1 - p$. A prospect is *riskless* if $p = 1$, otherwise it is *risky*.

Preferences over prospects are denoted as usual: the relation $\succeq$ denotes weak preference, $\succ$ denotes strict preference, and $\sim$ denotes indifference. Preferences over chronic health states correspond with preferences over riskless prospects. We assume throughout that prospects are *rank-ordered*, i.e., it is implicit in the notation $(p: (Q_1, T_1); (Q_2, T_2))$ that $(Q_1, T_1) \succeq (Q_2, T_2)$. *Expected utility* holds if there exists a function U from the set of chronic health states to the real numbers, called the *utility function*, such that prospects $(p: (Q_1, T_1); (Q_2, T_2))$ are evaluated by $pU(Q_1, T_1) + (1 - p)U(Q_2, T_2)$ and preferences and choices correspond with this evaluation.

Prospect theory deviates in three important respects from expected utility. First, carriers of value are gains and losses relative to a *reference point*. The location of the reference point is exogenously given and is not specified by prospect theory. We will denote the reference point as (Q_0, T_0). In the formal analysis of Kahneman and Tversky (1979), where there is only one fixed reference point, the reference point is assigned utility zero. In this paper, we will consider variations in the reference point and, hence, we do not follow this convention. A second deviation from expected utility is that people are more sensitive to losses than to corresponding gains, a phenomenon known as *loss aversion*. Third, people do not evaluate probabilities linearly, but weight probabilities; probability weighting for gains can be different from probability weighting for losses.

A prospect is *mixed* if it involves both a gain and a loss. A mixed prospect $(p: (Q_1, T_1); (Q_2, T_2))$, $(Q_1, T_1) \succ (Q_0, T_0) \succ (Q_2, T_2)$, $p \in (0, 1)$, is evaluated as

$$PT(p: (Q_1, T_1); (Q_2, T_2))$$
$$= U(Q_0, T_0) + w^+(p)(U(Q_1, T_1) - U(Q_0, T_0))$$
$$- \lambda w^-(1 - p)(U(Q_0, T_0) - U(Q_2, T_2)), \quad (1)$$

where PT is the function assigning utility to a prospect under prospect theory assumptions, w^+ and w^- are probability weighting functions for gains and losses, and λ is a loss aversion parameter. The *probability weighting functions* assign weights of 0 and 1 to probabilities of 0 and 1, respectively, and are strictly increasing over the interval $[0, 1]$. We separate loss aversion from utility because we consider varying reference points and we want to establish a link with expected utility. Therefore, the utility function U reflects the intrinsic utility of chronic health states. Our method for modelling loss aversion is similar to Shalev (2000) and Bleichrodt et al. (2001). In studies where the reference point is fixed, loss aversion is

often incorporated in the utility function (Kahneman and Tversky 1979, Tversky and Kahneman 1992). In (1), outcomes are evaluated as deviations from the reference point through terms $U(Q_1, T_1) - U(Q_0, T_0)$ so as to combine the psychology of prospect theory with the utility function U of expected utility.

If $(Q_1, T_1) \succeq (Q_2, T_2) \succeq (Q_0, T_0)$, then

$$PT(p: (Q_1, T_1); (Q_2, T_2))$$
$$= U(Q_0, T_0) + w^+(p)(U(Q_1, T_1) - U(Q_0, T_0))$$
$$+ (1 - w^+(p))(U(Q_2, T_2) - U(Q_0, T_0)). \quad (2)$$

If $(Q_0, T_0) \succeq (Q_1, T_1) \succeq (Q_2, T_2)$, then

$$PT(p: (Q_1, T_1); (Q_2, T_2))$$
$$= U(Q_0, T_0) - \lambda w^-(1 - p)(U(Q_0, T_0) - U(Q_2, T_2))$$
$$- \lambda(1 - w^-(1 - p))(U(Q_0, T_0) - U(Q_1, T_1)). \quad (3)$$

Equation (3) is best interpreted as the dual of (2) with $1 - w^-(1 - p)$ instead of $w^+(p)$ and a loss aversion parameter λ added to all utility differences.

We assume throughout that changes in the reference point leave the probability weighting functions w^+ and w^-, the utility function U, and the loss aversion parameter λ unchanged. Empirical studies have shown that the most common pattern for the probability weighting functions is inverse S-shaped, overweighting small probabilities and underweighting intermediate and large probabilities (Tversky and Kahneman 1992, Tversky and Fox 1995, Wu and Gonzalez 1996, Gonzalez and Wu 1999, Abdellaoui 2000, Bleichrodt and Pinto 2000). Tversky and Kahneman (1992) proposed the following one-parameter functional form for the probability weighting function:

$$w(p) = \frac{p^\gamma}{(p^\gamma + (1 - p)^\gamma)^{1/\gamma}}, \quad (4)$$

which has an inverse S-shape for γ between 0.27 and 1. Tversky and Kahneman (1992) found a median value for γ^+ (the parameter for gains) of 0.61 and for γ^- (the parameter for losses) of 0.69. Later studies found comparable median values for γ^+ and γ^- both for monetary (Wu and Gonzalez 1996, Abdellaoui 2000) and for health outcomes (Bleichrodt and Pinto 2000). These values imply that the probability for which the probability weighting functions change from overweighting probabilities to underweighting probabilities, i.e., the probability for which $w(p) = p$, lies between 0.30 and 0.40. Tversky and Kahneman (1992) also estimated the loss aversion coefficient and found a median value for λ equal to 2.25. A comparable value was obtained by Bleichrodt et al. (2001). Although median probability weighting and loss aversion parameter values were comparable across studies, all studies found wide variation in parameter estimates at the individual level.

Table 1   The Five Elicitation Methods Used

Method	Question
Riskless-risk methods	
Probability equivalence (PE)	$(Q, T) \sim (\boldsymbol{p}: (\text{FH}, T); \text{Death})$
Certainty equivalence (CE)	$(Q, \boldsymbol{T_{\text{ce}}}) \sim (p: (\text{FH}, T); \text{Death})$
Value equivalence (VE)	$(Q, T) \sim (p: (\text{FH}, \boldsymbol{T_{\text{ve}}}); \text{Death})$
Risk-risk methods	
Probability lottery equivalence (PLE)	$(0.35: (Q, T); \text{Death}) \sim (\boldsymbol{r}: (\text{FH}, T); \text{Death})$
Value lottery equivalence (VLE)	$(0.35: (Q, T); \text{Death}) \sim (0.35: (\text{FH}, \boldsymbol{T_{\text{vle}}}); \text{Death})$

Note. FH stands for "full health."

3.   Elicitation Methods

We used five methods to measure the utility of health states: three riskless-risk methods and two risk-risk methods. Table 1 shows the elicitation methods that we used; the response that was elicited is printed in bold. The health state whose utility is measured is denoted Q. The first three methods are riskless-risk methods. The PE method elicited the probability p that made a subject indifferent between (Q, T) for certain and a risky prospect giving (FH, T) with probability p and Death with probability $1 - p$, where FH stands for full health. The CE method elicited the duration T_{ce} that made a subject indifferent between (Q, T_{ce}) for certain and a risky prospect giving (FH, T) with probability p and Death with probability $1 - p$. The *value equivalence* (VE) method elicited the duration T_{ve} that made a subject indifferent between (Q, T) for certain and a risky prospect giving (FH, T_{ve}) with probability p and Death with probability $1 - p$. To enhance comparability between the three riskless-risk methods, the selected value of the *gauge duration T* was the same in all riskless-risk methods and the response p elicited in the PE method was also used in the CE and the VE methods.

The final two methods are risk-risk methods. The *probability lottery equivalence* (PLE) method elicited the probability r that made a subject indifferent between the risky prospect that gives (FH, T) with probability r and Death with probability $1 - r$ and the risky prospect that gives (Q, T) with probability 0.35 and Death with probability 0.65. The *value lottery equivalence* (VLE) method elicited the duration T_{vle} that made a subject indifferent between the risky prospect that gives (FH, T_{vle}) with probability 0.35 and Death with probability 0.65 and the risky prospect that gives (Q, T) with probability 0.35 and Death with probability 0.65. The gauge duration T was the same in the PLE and the VLE methods and was set equal to the value of T that was used in the riskless-risk methods. The use of probability 0.35, for which previous studies observed little median probability distortion, in the risk-risk methods does not entail that probability weighting plays no role in these methods as r would generally be lower than 0.35 and would be affected by

probability weighting according to previous empirical findings.

It is well known that indifference judgments tend to be affected by scale compatibility if the dimension on which indifference is established is changed. Scale compatibility predicts that the weight people assign to an attribute increases as this attribute is more compatible with the response scale used (Tversky et al. 1988; Delquié 1993, 1997). The effect of scale compatibility in the present experiment is ambiguous. For instance, in the PE method the response scale is probability and, consequently, scale compatibility predicts that people will focus on probability when evaluating prospects. However, people could either focus on the probability p of the good outcome of full health or on the probability $1 - p$ of the bad outcome of Death, and the direction of the bias due to scale compatibility depends on which probability people focus. We tried to control for compatibility effects by determining all indifferences through a choice-based task.

Another type of compatibility effect is *strategy compatibility*, according to which qualitative decision tasks, such as choice, are compatible with qualitative decision strategies, such as lexicographic ordering, and quantitative decision tasks, such as matching, are compatible with quantitative decision strategies (Fischer and Hawkins 1993). Prior research has shown that strategy compatibility is stronger than scale compatibility (Fischer and Hawkins 1993, Delquié 1997). Fischer et al. (1999) suggested that task goals play a central role in the construction of preferences. Specifically, they proposed that the prominent attribute of an alternative is weighted more heavily in response tasks whose perceived goal is to differentiate between alternatives than in tasks whose perceived goal is to equate alternatives. Neither strategy compatibility nor the task-goal hypothesis predict a bias in our results because we used the same decision task, the determination of indifferences through choices, in all five methods.

4.   Predictions

We assumed throughout that people prefer a longer life duration to a shorter one, both in full health

and in health state Q. Expected utility then predicts that, except for random error, we should observe that $T = T_{ce} = T_{ve}$.[1] The two risk-risk methods could not be directly compared with each other and with the riskless-risk methods because they involved different probabilities.

The predictions made by prospect theory depend on the location of the reference point. Hershey and Schoemaker (1985) and Bleichrodt et al. (2001) conjectured that in a utility elicitation task in which a subject compares two prospects and has to create an equivalence by varying a probability or outcome level of one prospect, he will take as reference level another outcome of the prospect that remains constant in the equivalence judgment. Their conjecture has been corroborated by empirical evidence (Stalmeier and Bezembinder 1999, Morrison 2000, Bleichrodt et al. 2001, Robinson et al. 2001). In the PE and in the VE, this argument implies that the reference point is (Q, T). In the CE, where the risky prospect remains constant, the reference point is either (FH, T) or Death. The data in Bleichrodt et al. (2001) suggested that people take Death as their reference point in the CE questions.

Consequently, we would expect that $T = T_{ve}$ under prospect theory because the reference point is the same in the PE and in the VE. However, T_{ce} may well differ from T. Under the above conjecture, the PE and VE compare mixed prospects, whereas the CE compares either two prospects involving only gains or two prospects involving only losses. It is well known that, due to loss aversion, people are more risk averse for mixed prospects than for prospects involving only gains or only losses (Payne et al. 1980, 1981) and, hence, we expect that $T_{ce} > T$. Again, because the risk-risk methods use different probabilities they do not yield specific predictions unless additional assumptions are made.

We obtained more conclusive tests of expected utility and prospect theory by computing health state utilities. To be able to compute health state utilities, we assumed in all models *multiplicativity* of U: $U(Q, T) = H(Q)L(T)$, where H and L are real-valued utility functions over the set of health states and the set of life durations, respectively. Throughout, we used the scaling $U(\text{Death}) = 0$ and $H(\text{FH}) = 1$.

Empirical support for multiplicativity was obtained by Miyamoto and Eraker (1988), Doctor et al. (2004), and Bleichrodt and Pinto (2005). Multiplicativity can be tested by comparing the answers to the PE questions for different gauge durations with each other and by comparing the answers to the PLE questions

Table 2   **Utilities Under Expected Utility (EU) and Prospect Theory (PT)**

	PE	CE	VE	PLE	VLE
EU-linear	p	$p\dfrac{T}{T_{ce}}$	$p\dfrac{T_{ve}}{T}$	$\dfrac{r}{0.35}$	$\dfrac{T_{vle}}{T}$
EU-power	p	$p\left(\dfrac{T}{T_{ce}}\right)^{\beta}$	$p\left(\dfrac{T_{ve}}{T}\right)^{\beta}$	$\dfrac{r}{0.35}$	$\left(\dfrac{T_{vle}}{T}\right)^{\beta}$
PT	A	RP Death: $w^+(p)\left(\dfrac{T}{T_{ce}}\right)^{\beta}$ RP(FH, T): $(1-w^-(1-p))\left(\dfrac{T}{T_{ce}}\right)^{\beta}$	$A\left(\dfrac{T_{ve}}{T}\right)^{\beta}$	RP Death: $\dfrac{w^+(r)}{w^+(0.35)}$ RP(Q,T): B RP(FH, T): $\dfrac{1-w^-(1-r)}{1-w^-(0.65)}$	RP Death: $\left(\dfrac{T_{vle}}{T}\right)^{\beta}$ RP(Q,T): $\left(\dfrac{T_{vle}}{T}\right)^{\beta}$

Note. RP stands for reference point. $A = w^+(p)/(w^+(p) + \lambda w^-(1 - p))$. $B = w^+(r)/(w^+(r) + \lambda(w^-(1 - r) - w^-(0.65)))$.

for different gauge durations with each other. Multiplicativity implies that, except for random response error, we should find the same probabilities p in the different PE questions and the same probabilities r in the different PLE questions.

To be able to compute $H(Q)$ from the responses to the CE, VE, and VLE questions, we had to assume a specific form for $L(T)$. We first assumed that L is linear in which case $U(Q, T)$ is equal to the quality-adjusted life-years (QALY) model, the most widely used model in medical decision analysis. The first row of Table 2 shows the expression for $H(Q)$ under expected utility with linear utility. There is little empirical support for the assumption that utility is linear in life duration; an exception is Doctor et al. (2004). We, therefore, subsequently assumed that L is a power function: $L(T) = T^{\beta}$. The power function is often used in decision analysis and several studies have observed that it yields a good fit in the health domain (e.g. Pliskin et al. 1980, Miyamoto and Eraker 1985, Stiggelbout et al. 1994, Cher et al. 1997). The second row of Table 2 displays the effect on $H(Q)$ of replacing the assumption of linear utility by the assumption of power utility. The table shows that the PE and the PLE methods are not affected by the choice of L.

The final part of Table 2 shows $H(Q)$ under prospect theory with power utility for life duration. To be able to evaluate the five elicitation methods under prospect theory, we had to specify the location of the reference point for each method. Here we followed the suggestions of Hershey and Schoemaker (1985) and Bleichrodt et al. (2001) and took (Q, T) as the reference point in the PE and in the VE and either (FH, T) or Death in the CE. In the PLE, we took either (FH, T), or (Q, T) or Death as the reference point. In the VLE, (FH, T_{vle}) was implausible as a reference

point because T_{vle} was varied to create an equivalence and we, therefore, only analyzed the data for reference points (Q, T) and Death. To compute the probability weights, we assumed Tversky and Kahneman's (1992) functional form (4) with γ^+ denoting the probability weighting parameter for gains and γ^- the probability weighting parameter for losses.

5. Experiment

5.1. Background

The subjects were 65 economics students (aged between 22 and 29) from the University of Murcia. They were paid €36 to participate in five experimental sessions, each lasting approximately one hour. In each experimental session, a different elicitation method was administered. The experiment was carried out in small group sessions with at most six subjects per group. The sessions were separated by at least one week. Prior to the actual experiment, the questionnaire was tested in several pilot sessions using university staff as subjects.

We elicited the utility of two health states. The health states were described through the EuroQol system, a widely used instrument to describe health states in medical research. The description of the health states is given in Table 3. Throughout the experiment, the health states were labelled as A and B. Preferences were elicited through a sequence of choices in which we only varied the parameter that we sought to elicit across choices. We always started with parameter values for which one of the alternatives was clearly better than the other and then "zoomed in" on the parameter value for which subjects were indifferent between the alternatives. Full details on the elicitation procedure can be found in the electronic companion.[2] We used a choice-based procedure because empirical evidence suggests that determining indifferences through choices leads to fewer inconsistencies than determining indifferences through matching (Bostic et al. 1990).[3]

5.2. Details

Recruitment of subjects took place one week before the actual experiment started. At the recruitment, subjects received information about the experiment and were asked to read the descriptions of the two health states. In addition, the subjects were handed a practice question on the PE method. They were asked to

Table 3 The Description of Health States A and B

Health state A	Health state B
• Some problems walking about	• Some problems walking about
• Some problems performing self-care activities (e.g. eating, washing, dressing)	• Some problems performing self-care activities (e.g. eating, washing, dressing)
• No problems performing usual activities (e.g. work, study, family or leisure activities)	• Unable to perform usual activities (e.g. work, study, family or leisure activities)
• Moderate pain or discomfort	• Moderate pain or discomfort
• Moderately anxious or depressed	• Moderately anxious or depressed

answer this practice question at home. This procedure was intended to familiarize them with the PE method. Prior to the start of the first experimental session, during which the PE method was administered, the subjects were asked to explain their answer to the practice question. When we were not convinced that a subject understood the task, we explained it again until we were convinced that he understood the task. The same procedure was repeated for each of the remaining experimental sessions. The subjects received a practice question to take home showing the method that would be administered in the next session, and they had to explain their answer to the question before the actual experiment started.[4] To motivate the subjects, we told them at the start of the first experimental session that their answers were important for health policy to determine priorities between medical treatments.

The order in which the methods were administered was: first session PE, second CE, third PLE, fourth VE, and fifth VLE. The experiment was part of a larger experiment. We assumed that the presence of the other experimental tasks and the delay of at least one week between the sessions made it unlikely that the subjects would recall their previous answers or would note the relationship between the sessions.

At the beginning of each experimental session, instructions were read aloud and an additional practice question was given. We asked six questions per method by combining each health state (A and B) with three values for the gauge duration T: 13, 24, and 38 years. We used life durations substantially lower than the subjects' life expectancy to avoid perception problems: subjects may find it hard to perceive living for very long durations which exceed their life expectancy. To avoid order effects, we varied the order in which the different questions were asked within a section. To minimize response errors, the subjects had to confirm the elicited indifference value after each question.

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[2] An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

[3] It should be noted that our method for determining indifferences through choice differed from the PEST procedure, the psychometric estimation procedure that Bostic et al. used. It is, however, plausible that the advantages of using choice also accrue to the present study.

[4] The questionnaire that we used can be found in the electronic companion.

We also determined through a choice-based procedure the life duration T' that made a subject indifferent between T years in health state A and T' years in full health, for T equal to 13, 24, and 38 years. The same question was asked for health state B. These questions were included to test the appropriateness of assuming power utility for life duration. Under power utility, the ratio T'/T should be constant. Also, in the presence of multiplicativity, the condition that the ratio T'/T is constant implies that the utility for life duration must be a power function (Doctor and Miyamoto 2003). These questions were asked in the second experimental session.

5.3. Analysis

The nonparametric Friedman test was used to test for significance of utility differences among the five methods and for significance of differences between T, T_{ce}, and T_{ve}. When the hypothesis of equality was rejected by the Friedman test, we performed multiple pairwise comparisons by the Wilcoxon signed-rank test. The Friedman test was also used to test for the appropriateness of assuming multiplicativity (p should be the same in the three PE questions and r should be the same in the three PLE questions) and the power function (T'/T should be constant; see above). In all tests, we used a significance level of 1% to control for experimentwise Type I error, the phenomenon that when many tests on a given level are performed, some will be significant by chance (multiple significance testing).

A distribution-free algorithm was used to determine the optimal values of the parameters in expected utility and prospect theory for each subject separately based on the utilities elicited by the five methods. We performed this analysis for each of the six health state-gauge duration pairs. We started by setting each parameter equal to one, the case corresponding to expected utility with linear utility for life duration. Then, we searched for the values of the parameters that minimized the sum of squared differences between the elicited utilities.[5] We varied β between 0.05 and 2, γ^+ and γ^- between 0.25 and 2, and λ between 0.25 and 4. Using wider bounds occasionally caused the program to choose extreme and implausible values so that all utilities were close to zero. The range of parameters used includes the estimates from the existing empirical literature. The optimal parameters were determined with an accuracy of 0.01.

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[5] To examine the sensitivity of the results to outliers, we also determined the parameters that minimized the sum of absolute differences. There were only small differences between the two sets of estimates.

Table 4   **Median Responses**

T	p	T_{ce}	T_{ve}	r	T_{vle}
Health state A					
13 years	0.58	17	17	0.16	6
	(0.54–0.63)	(14–22)	(11–21)	(0.12–0.19)	(4–6)
24 years	0.68	27	28	0.17	11
	(0.65–0.73)	(23–35)	(22–36)	(0.12–0.19)	(9–13)
38 years	0.72	42	46	0.17	18
	(0.68–0.77)	(38–48)	(41–48)	(0.13–0.18)	(16–19)
Health state B					
13 years	0.49	18	14	0.13	4
	(0.45–0.55)	(13–25)	(10–18)	(0.11–0.16)	(3–6)
24 years	0.57	31	21	0.13	8
	(0.51–0.65)	(24–36)	(14–38)	(0.11–0.16)	(7–11)
38 years	0.59	44	43	0.13	14
	(0.50–0.68)	(39–48)	(32–51)	(0.11–0.14)	(12–17)

Note. Interquartile ranges are in parentheses.

6. Results

6.1. Preliminaries

Two subjects were excluded from the analyses of health state A and 19 from the analyses of health state B because their choices implied that they did not always prefer more life-years to less. This left 63 and 46 subjects in the analyses of health states A and B, respectively. More of the subjects had to be excluded for health state B, because B is a worse health state than A. The worse a health state, the more likely there is a duration for which the subjects do not prefer additional life-years. The excluded subjects were those with the lowest utilities. The fact that more of the subjects were excluded for health state B than for health state A will not bias our conclusions because they are not based on comparisons between the utilities for health states A and B.

Table 4 shows the median responses and in parentheses the interquartile range of the responses. The mean responses were similar to the medians. Because health state A is better than health state B, we should observe higher values of p, r, and T_{vle} for health state A. All the subjects satisfied this consistency requirement. Note that r was much lower than 0.35, and was within a range for which previous studies found overweighting of probabilities. Hence, probability weighting could affect the PLE questions.

Contrary to the predictions of expected utility, we could reject equality of T, T_{ce}, and T_{ve} for both health states and for all three gauge durations ($p < 0.01$). The finding that T_{ce} significantly exceeded T ($p < 0.001$ in all pairwise comparisons) is consistent under prospect theory with more risk aversion for mixed prospects. Contrary to prospect theory, we also found that T_{ve} generally exceeded T with the exception of health state B with the gauge duration T equal to 24 years. The difference was always significant for health

state A ($p < 0.001$). For health state B, the difference was marginally significant for $T = 13$ years and $T = 38$ years ($p = 0.035$ and $p = 0.025$) and not significant for $T = 24$ years ($p = 0.650$).

The tests of multiplicativity yielded mixed results. For both health states, we could reject the hypothesis that the probabilities in the three PE questions were equal ($p < 0.01$ in both tests). However, we could not reject the hypothesis that the probabilities in the three PLE questions were equal ($p = 0.148$ for health state A and $p = 0.085$ for health state B). The tests of the appropriateness of using power utility for life duration yielded positive results: for both health states, we could not reject the hypothesis that the ratio T'/T was constant ($p = 0.565$ for health state A, $p = 0.085$ for health state B).

6.2. Main Findings

Figure 1 shows the median utilities under expected utility with linear utility for life duration. For both health states and for all gauge durations, we found significant differences between the five methods ($p < 0.001$ in all tests). The difference between the utilities was generally considerable with a maximum value of 0.39 (VE − VLE for health state A and gauge duration 38 years). The typical pattern was VE > PE > CE > PLE > VLE. The differences between PLE and VLE were, however, not significant. With few exceptions, all other paired differences were significant. Hence, we found that riskless-risk methods yielded inconsistent results, that riskless-risk methods led to higher utilities than risk-risk methods, and that risk-risk methods led to consistent results under expected utility with linear utility for life duration.

Table 5 reports the medians of the individual estimates of the optimal parameters under expected utility and prospect theory for each health state-gauge duration pair separately based on the utilities elicited by the five methods. The first row of Table 5 shows the results for expected utility with power utility. The estimates reflect a substantial degree of concavity of the utility function for life duration. However, the systematic differences between the five methods could not be explained by the assumed linearity of the utility for life duration only. We could clearly reject the hypothesis that the five methods yield the same utilities under expected utility with the optimal power coefficients ($p < 0.001$ for both health states and for all gauge durations). The consistency between PLE and VLE that we observed under expected utility with linear utility for life duration no longer holds: all differences between PLE and VLE were significant under expected utility with power utility for life duration.

The other rows of Table 5 display the medians of the individual parameter estimates for prospect theory where the reference point in the CE, PLE, and

Figure 1   Median Utilities Under Expected Utility with Linear Utility for Life Duration
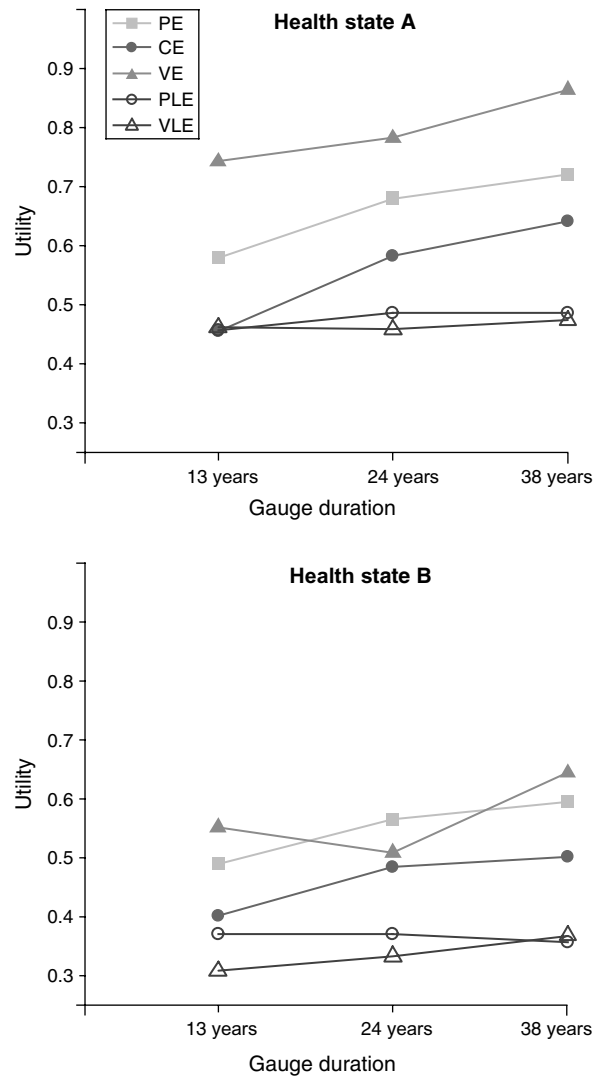
VLE methods is Death. We also analyzed the data under the other possible reference points, but these did not lead to smaller differences between the methods and the convergence, and fit at the individual

Table 5   Medians of the Individual Parameter Estimates

	Duration					
	Health state A			Health state B		
Model	13	24	38	13	24	38
EU-power						
β	0.46	0.45	0.48	0.60	0.53	0.60
Prospect theory						
β	0.86	0.73	0.65	0.78	0.65	0.65
λ	2.13	2.00	1.84	1.83	1.77	1.53
γ^+	0.93	0.79	0.73	0.95	0.84	0.77
γ^-	0.80	0.80	0.80	0.90	1.20	0.80

level was far worse. Compared with other studies, we observed less distortion in probability weighting and less loss aversion. The observation of less probability weighting is not consistent with Rottenstreich and Hsee (2001), who found that the probability weighting function is more curved for "affect-rich" outcomes, such as health, compared to "affect poor" outcomes, like money. Utility for life duration is concave; the estimates for the power coefficient agree with the findings from previous studies on the utility for life duration under nonexpected utility. There was considerable variation in the estimates at the individual level. The mean lengths of the interquartile range were 0.498, 1.509, 0.322, and 0.400 for β, λ, γ^+, and γ^-, respectively.

We further examined the data under prospect theory with the probability weighting and loss aversion parameters obtained by Tversky and Kahneman (1992): $\gamma^+ = 0.61$, $\gamma^- = 0.69$, and $\lambda = 2.25$. We analyzed this case because Bleichrodt et al. (2001) were able to remove all systematic discrepancies between PE and CE utilities using Tversky and Kahneman's values.

Neither of the two versions of prospect theory that we examined could fully explain the data. The differences between the five methods were significant for both versions ($p < 0.01$) except for health state B under prospect theory with the estimated optimal parameters where the differences between the methods were only marginally significant ($p = 0.040$, $p = 0.036$, and $p = 0.034$ for gauge durations 13 years, 24 years, and 38 years, respectively).

Table 6 shows the number of significant pairwise differences between the five elicitation methods.[6] For each gauge duration, there were 10 comparisons between methods (PE versus CE, PE versus VE, etc.). The table shows that there were many inconsistencies under expected utility with linear utility for life duration. Using power utility instead of linear utility did not improve the performance of expected utility. Prospect theory with the probability weighting and loss aversion parameters obtained by Tversky and Kahneman (1992) performed better, even though many differences remained significant.[7] The number of significant differences was lowest under prospect theory with the optimal parameters. The performance of prospect theory with the optimal parameters was particularly good for health state B. Of course, in interpreting these results one should keep in mind that prospect theory with the optimal parameters had more degrees of freedom than the other theories.

[6] Tables with all the *p*-values from the Wilcoxon tests are provided in the electronic companion.

[7] Data in the table for prospect theory with the Tversky–Kahneman parameters are under linear utility for life duration. Similar data were obtained when we allowed for curved utility for life duration.

Table 6 Number of Significant Pairwise Differences Between Methods Based on Median Parameters and a Significance Level of 1%

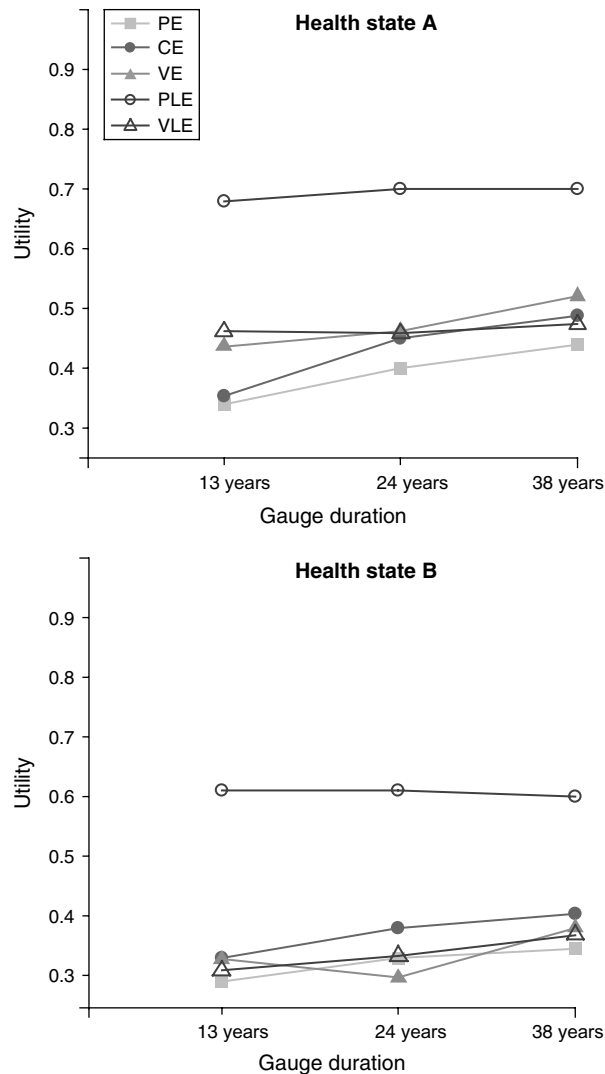|	Duration					
	Health state A			Health state B		
Model	13	24	38	13	24	38
EU-linear	7	7	9	6	7	8
EU-power	9	8	9	6	6	7
Prospect theory TK	6	7	6	4	4	6
Prospect theory opt.	3	4	4	1	2	1

Note. Prospect theory TK stands for prospect theory with the probability weighting and loss aversion parameters obtained by Tversky and Kahneman (1992).

Figure 2 shows the results under prospect theory with Tversky and Kahneman's (1992) values and linear utility for life duration. The figure shows that the main problem is that PLE was too high. As can be seen from Table 2, this problem cannot be solved by allowing for utility curvature because neither the PE nor the PLE are affected by the assumed form of the utility for life duration and, hence, their discrepancy will remain after correction for utility curvature. Under prospect theory, the difference between the PLE and the other methods can only be explained by a difference in the degree of probability weighting in our study as compared with Tversky and Kahneman (1992). Recall from Table 2 that the utility according to the PLE was equal to $w^+(r)/w^+(0.35)$. Because the PLE utilities were too high compared with the utilities elicited through the other four methods, this ratio was too high when Tversky and Kahneman's parameter values were used. Given that the response r in the PLE was generally between 0.10 and 0.20, our data suggest less overweighting of probabilities in that range than observed by Tversky and Kahneman (1992) assuming that there is comparable (absence of) probability weighting around 0.35.

Figure 2 also shows that prospect theory could rather well explain the differences between the PE, CE, VE, and VLE except that for health state A the PE was a bit too low. In these four methods, the probabilities involved generally exceeded 0.35, suggesting comparable probability weighting in the range [0.35, 1] as found by Tversky and Kahneman (1992). The finding that CE generally exceeded PE is in contrast with Bleichrodt et al. (2001). The difference between PE and CE was significant for health state A and gauge durations 24 years and 38 years and for health state B and gauge duration 38 years.

Figure 3 shows the median utilities under prospect theory with the optimal parameters. In general, the utilities were close; the exception is that for health state A the PE was too low. For health state B, there are no systematic differences between the five methods. The significant ($p < 0.01$) upward trends in PE,

Figure 2   **Median Utilities Under Prospect Theory with Tversky and Kahneman's (1992) Values and Linear Utility for Life Duration**

Figure 3   **Median Utilities Under Prospect Theory with Optimal Parameters**
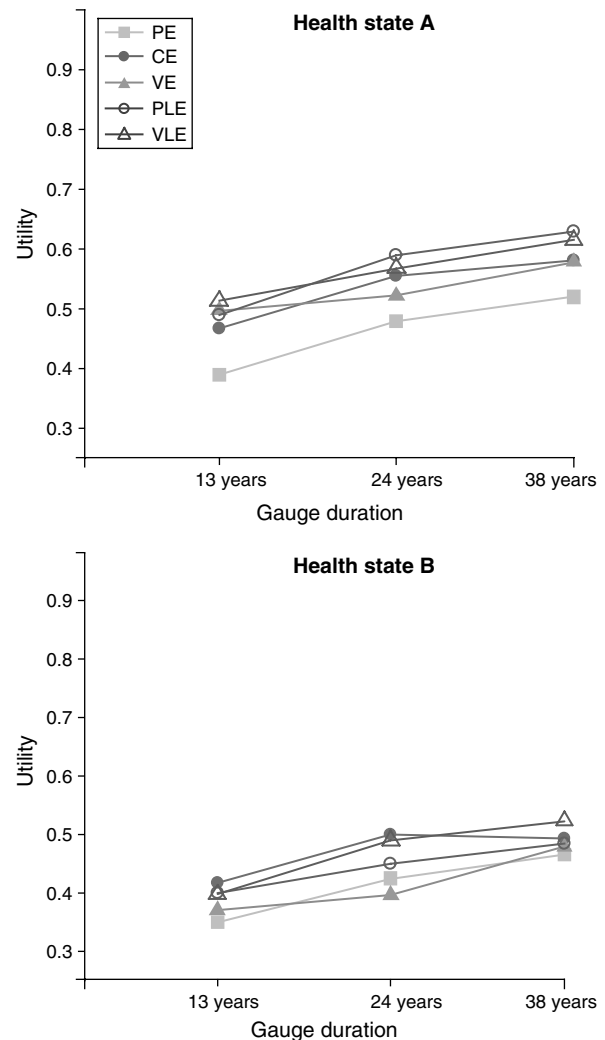
PLE, and VLE utilities can be explained by the differences in the optimal parameter estimates for the three gauge durations. In particular, the decrease in the power coefficients across the gauge durations leads to an increase in the VLE utilities, the decrease in loss aversion leads to higher PE utilities, and the decrease in γ^+ leads to higher PE and PLE utilities. The differences in parameter estimates have countervailing effects on the CE and VE utilities, which explains why the trends in these measures are not monotonic.

Finally, we examined the fit of the different versions of expected utility and prospect theory by imposing on each individual the median optimal parameters for each health state-gauge duration pair and by then examining which model yielded the lowest sum of squared errors between the five assessment methods. Table 7 reports the proportion of subjects for whom each model provided a superior fit to the data. The

table shows that for most subjects, prospect theory with the optimal parameters was most consistent with their data. These findings are not distorted by differences in degrees of freedom between the models because we imposed the median preferences on each subject.[8]

6.3. Auxiliary Analyses

Although prospect theory was more consistent with the data than expected utility, it could not entirely explain them and, hence, we also examined the data under four other nonexpected utility models. The models we considered were rank-dependent utility

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[8] There is a small caveat here. Because the median optimal parameters varied across health state-gauge duration pairs, expected utility with power utility and prospect theory with the optimal parameters had some additional flexibility compared with expected utility with linear utility and prospect theory with Tversky and Kahneman's (1992) parameters.

Table 7 Proportion of Individuals for Whom a Particular Model Fitted Best in Terms of the Sum of Squared Residuals Based on the Median Parameter Estimates

Health state	Model			
	EU linear	EU power	PT TK	PT opt
A, 13 years	12.7	38.1	7.9	41.3
A, 24 years	12.7	19.0	15.9	52.4
A, 38 years	7.9	9.5	12.7	69.8
B, 13 years	10.9	28.3	2.2	58.7
B, 24 years	4.3	26.1	10.9	58.7
B, 38 years	0	13.0	6.5	80.5

(Quiggin 1981), Gul's (1991) theory of disappointment aversion, and two recently proposed gambling effect models (Bleichrodt and Schmidt 2002, Diecidue et al. 2004). Rank-dependent utility is the special case of prospect theory where there is no loss aversion, i.e., Equation (2). Disappointment aversion is the special case of rank-dependent utility where the probability weighting function is equal to $p/(1+(1-p)\delta)$. The parameter $\delta \in (-1, \infty)$ reflects disappointment aversion. The two gambling effect models deviate from expected utility by assuming that there is not one utility function over outcomes but two. In the model of Diecidue et al. (2004), preferences are *prospect-dependent*: if a prospect is risky, its outcomes are evaluated by a utility function U; if it is riskless, its outcomes are evaluated by a utility function V. In the model of Bleichrodt and Schmidt (2002), preferences are *context-dependent*: when both prospects in a comparison are risky, their outcomes are evaluated by a utility function U; otherwise, the outcomes of both prospects are evaluated by a utility function V. An interesting property of the gambling effect models is that they predict that under expected utility riskless-risk methods lead to higher utilities than risk-risk methods and that expected utility will give consistent results in risk-risk methods, two predictions that were confirmed by our data.

None of the four models fitted the data as well as prospect theory with the optimal parameters. The differences between the five methods were significant for all theories ($p < 0.01$ in all cases), except for health state B and gauge duration 24 years in the model of Bleichrodt and Schmidt (2002) ($p = 0.084$). The number of significant pairwise differences between the five methods was in all models higher than under prospect theory with the optimal parameters, although this number was also low for health state B under Bleichrodt and Schmidt's (2002) gambling effect model. Finally, prospect theory with the optimal parameters was clearly the best-fitting model when we imposed the median optimal estimates on each subject and then examined for each subject which

model provided the superior fit. Details on the operationalization, the formulas for $H(Q)$, the parameter estimates, the results from the auxiliary analyses and figures for rank-dependent utility, disappointment aversion, and the two gambling effect models are provided in the electronic companion.

7. Discussion

Our findings confirm that methods that are equivalent according to expected utility produce systematically different results. The data suggest that evaluating riskless-risk methods through expected utility leads to utilities that are too high. We found no significant differences between risk-risk methods under expected utility (when the utility for life duration is linear) and our data seem to add to the evidence that violations of expected utility primarily occur when one of the prospects under evaluation is riskless. Of the nonexpected utility models we studied, prospect theory with parameters tailored to the specific sample was most consistent with the data. Prospect theory could explain the systematic discrepancies between the riskless-risk methods. The finding that the two risk-risk methods yielded comparable results under expected utility is, however, harder to reconcile with prospect theory. The common observation is that probabilities are overweighting in the interval [0.10, 0.20] and, consequently, PLE should be lower than VLE when responses are analyzed assuming expected utility. Our finding suggests that there was little overweighting of probabilities in the interval [0.10, 0.20].

It may be too optimistic to assume that one single model could explain all data. After all, any theory is necessarily incomplete and restricted in its scope (Payne et al. 1999). The number of deviations from expected utility that we considered was limited and other forms of bias will most likely have affected people's responses. In particular, we ignored the impact of scale compatibility. Scale compatibility could explain, for instance, why T_{ve}, the response to the VE question, generally exceeded T, a finding that none of the theories considered could explain.

To operationalize prospect theory, we had to make assumptions about the location of the reference point. Empirical evidence seems to support our assumptions about the location of the reference point in the riskless-risk methods. No evidence exists about the location of the reference point in the risk-risk questions. Here we extended the conjectures of Hershey and Schoemaker (1985) and Bleichrodt et al. (2001). The formation of a reference point may, however, be more complicated when both prospects are risky. For example, people may take a risky prospect as their reference point rather than a single outcome. It

might be that in the PLE method, where the probability r was determined that made a subject indifferent between the risky prospects (r: (FH, T); Death) and (0.35: (Q, T); Death), the reference point for the best outcome of the prospects was (Q, T), but for the worst outcomes it was Death. It is not clear how to cover such a situation. In particular, it is not clear how to model probability weighting when the reference point is a risky prospect. Sugden (2003) presented such an extension for the case where people do not weight probabilities.

Our study was motivated by the idea that if five measurement methods yield consistent results under a particular preference theory, then this theory is supported as a descriptive theory of decision under risk. There does not exist a gold standard for utility, however, and reconciliation between different measurements of utility suggests but does not prove that a particular preference theory is closer to people's true preferences. Whether other ways of assessing the descriptive validity of preference theories, e.g., by asking direct isolated choices, will produce similar conclusions is obviously an open question.

A drawback of using health outcomes is that we had to assume multiplicativity to be able to compute utilities. We observed mixed evidence on multiplicativity. It should be noted that even when multiplicativity was violated, we could still compare the PE utilities with the PLE utilities. In that case, we compared the utilities $U(Q, T)$ under PE and PLE rather than the utilities $H(Q)$. Our conclusions were not affected when we only compared the PE with the PLE. Thus, the assumption of multiplicativity does not seem to be critical in the findings of our study.

Because we elicited preferences over health, the outcomes in our study had to be hypothetical. Several studies have addressed the question whether response patterns differ between questions with hypothetical outcomes and questions with real outcomes; see Camerer and Hogarth (1999) and Hertwig and Ortmann (2001) for extensive reviews. These studies used moderate monetary amounts as outcomes. The general conclusion from these studies is that the effect of real incentives varies across decision tasks. For the kind of tasks that we asked our subjects to perform, the determination of indifferences between binary prospects through choices, there appears to be no systematic difference in the general pattern of responses, although real incentives tend to reduce data variability.

Another problem with using health outcomes could be that subjects had problems imagining the health states. We used a common way to describe health states in medical research, but these descriptions are admittedly abstract and may have caused problems of imagination and, consequently, unreliable answers.

The use of students as subjects may limit the generalizability of our findings. Empirical evidence on health utility measurement has suggested, however, that there are no systematic differences in the patterns of responses obtained using convenience samples and those obtained using representative samples from the general population. For a review, see de Wit et al. (2000). A final limitation of our study is that we used a fixed order in which the five methods were administered. This may have affected the results and it would have been better to change the order in which the methods were administered. We could exclude, however, the possibility that people behaved more in agreement with expected utility in later sessions, contradicting the hypothesis that more experience leads to fewer violations of expected utility.

8. Implications

Let us finally discuss the implications of our findings for decision analysis practice. Our findings corroborate and extend earlier findings that different assessment procedures yield inconsistent results under expected utility. The differences are generally substantial and can be expected to affect the outcomes of practical decision analyses. In particular, our findings suggest that the common practice in (medical) decision analysis to measure utilities under risk by PE or CE methods and to evaluate the responses through expected utility will lead to utilities that are inconsistent and biased upwards. The best way to solve these inconsistencies is to adopt a constructive preference approach and to solve the inconsistencies in an interactive process. Often such an approach is not possible, however. For example, in medical decision analysis, utilities are commonly measured by medical staff who lack the time and training to solve the inconsistencies in utility measurement. Then, the results of this paper may be useful.

At the aggregate level, our results show that expected utility should not be used to evaluate riskless-risk methods because the resulting utilities will be too high. Instead, these methods should be evaluated by prospect theory. The parameters found by Tversky and Kahneman (1992) performed well for riskless-risk methods although our data suggest somewhat less distortion in probability weighting and less loss aversion than implied by Tversky and Kahneman's (1992) parameters. For risk-risk methods, we found no inconsistencies under expected utility. The absence of inconsistencies under expected utility can be explained by the virtual absence of overweighting of probabilities in the interval [0.10, 0.20], however, and we therefore feel that we cannot recommend using expected utility to evaluate risk-risk methods on the basis of our findings alone.

At the individual level, the picture is more complex because we found considerable variation in optimal parameter estimates. Here the best strategy seems to use several assessment methods simultaneously. Our data can help in the selection of these methods. For example, in medical decision analysis, the PE and the PLE may be good choices because these require no assumptions about utility for life duration. Alternatively, the CE, PLE, and VLE may be selected because our data suggest that these are not affected by loss aversion. Finally, an argument in favor of using the VE and the VLE can be that these methods yielded the largest differences in elicited utilities under expected utility. If the selected assessment procedures give different results, then the first step is to verify whether these differences affect the recommendations of the decision analysis. If so, and assuming that the constructive preference approach is not feasible, then the results of this paper can be useful in determining which theory is most consistent with a client's responses. The paper has derived and displayed patterns between assessment procedures and checking these patterns will improve the representation of the client's true preferences. If the patterns agree with the patterns observed in this paper, then our individual results (Table 7) suggest that it is better to evaluate the client's responses by prospect theory with the parameters that we obtained than by expected utility even though this implies imposing the same preferences on all clients and ignoring the substantial variation in individual estimates.

9. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

Appendix. Derivation of the Formulas in Table 2

Throughout we assume multiplicativity and power utility for life duration $U(Q, T) = H(Q)T^\beta$ and we use the scaling $H(\text{FH}) = 1$ and $U(\text{Death}) = 0$. Expected utility with linear utility is the special case of expected utility with power utility where $\beta = 1$.

PE Method

The indifference $(Q, T) \sim (p: (\text{FH}, T); \text{Death})$ yields under expected utility $H(Q)T^\beta = pT^\beta$ or $H(Q) = p$. Under prospect theory, (1) and (3) yield $H(Q)T^\beta = H(Q)T^\beta + w^+(p)(T^\beta - H(Q)T^\beta) - \lambda w^-(1 - p)H(Q)T^\beta$. Rearranging and deleting the common term T^β gives $H(Q) = w^+(p)/(w^+(p) + \lambda w^-(1-p))$.

CE Method

The indifference $(Q, T_{ce}) \sim (p: (\text{FH}, T); \text{Death})$ yields under expected utility $H(Q)T_{ce}^\beta = pT^\beta$ and thus $H(Q) = p(T/T_{ce})^\beta$. Under prospect theory with reference point Death, $H(Q)T_{ce}^\beta = w^+(p)T^\beta$ and thus $H(Q) = w^+(p)(T/T_{ce})^\beta$. If the reference point is (FH, T), (3) gives $T^\beta - \lambda(T^\beta - H(Q)T_{ce}^\beta) = T^\beta - \lambda w^-(1-p)T^\beta$. Deleting common terms and rearranging gives $H(Q) = (1 - w^-(1-p))(T/T_{ce})^\beta$.

VE Method

The indifference $(Q, T) \sim (p: (\text{FH}, T_{ve}); \text{Death})$ yields under expected utility $H(Q)T^\beta = pT_{ve}^\beta$ and thus $H(Q) = p(T_{ve}/T)^\beta$. Prospect theory with reference point (Q, T) gives by (1), $H(Q)T^\beta = H(Q)T^\beta + w^+(p)(T_{ve}^\beta - H(Q)T^\beta) - \lambda w^-(1 - p)H(Q)T^\beta$. Rearranging gives $H(Q) = w^+(p)/(w^+(p) + \lambda w^-(1-p))(T_{ve}/T)^\beta$.

PLE Method

The indifference $(0.35: (Q, T); \text{Death}) \sim (r: (\text{FH}, T); \text{Death})$ yields under expected utility $0.35H(Q)T^\beta = rT^\beta$ or $H(Q) = r/0.35$. Under prospect theory with reference point Death, $w^+(0.35)H(Q)T^\beta = w^+(r)T^\beta$ or $H(Q) = w^+(r)/w^+(0.35)$. If the reference point is (Q, T), (1) and (3) give $H(Q)T^\beta - \lambda w^-(0.65)H(Q)T^\beta = H(Q)T^\beta + w^+(r)(T^\beta - H(Q)T^\beta - \lambda w^-(1 - r)H(Q)T^\beta$. Rearranging gives $H(Q) = w^+(r)/(w^+(r) + \lambda(w^-(1 - r) - w^-(0.65)))$. If the reference point is (FH, T), (3) gives $T^\beta - \lambda w^-(1 - r)T^\beta = T^\beta - \lambda w^-(0.65)T^\beta - \lambda(1 - w^-(0.65))(T^\beta - H(Q)T^\beta)$. Rearranging gives $H(Q) = (1 - w^-(1 - r))/(1 - w^-(0.65))$.

VLE Method

The indifference $(0.35: (Q, T); \text{Death}) \sim (0.35: (\text{FH}, T_{vle}); \text{Death})$ yields under expected utility $0.35H(Q)T^\beta = 0.35T_{vle}^\beta$ or $H(Q) = (T_{vle}/T)^\beta$. Prospect theory with reference point Death gives $w^+(0.35)H(Q)T^\beta = w^+(0.35)T_{vle}^\beta$ or $H(Q) = (T_{vle}/T)^\beta$. If the reference point is (Q, T), (1) and (3) give $H(Q)T^\beta - \lambda w^-(0.65)H(Q)T^\beta = H(Q)T^\beta + w^+(0.35)(T_{vle}^\beta - H(Q)T^\beta) - \lambda w^-(0.65)H(Q)T^\beta$. Deleting common terms and rearranging gives $H(Q) = (T_{vle}/T)^\beta$.

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