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Survival risks, intertemporal consumption, and insurance: The case of distorted probabilities

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Abstract

This paper explores how to evaluate changes in survival probabilities when people do not process probabilities linearly, as is commonly assumed in the literature, but distort probabilities. We show that the valuation of risks to life depends critically on two parameters: the elasticity of the probability weighting function and the elasticity of the utility function with respect to future consumption. Using estimates from the empirical literature we derive that the bias of erroneously ignoring probability distortion in general leads to cost–benefit ratios that are too high and that generate too much priority for programs that save young lives.

JEL classification: D61; D81; D91IB11; IE26; IE32

Keywords: Intertemporal consumption; Survival risks; Cost–benefit analysis; Probability distortion; Non-expected utility

1. Introduction

Cost–benefit analyses of policies that cause changes in risks to human life require that a monetary value be attached to the changes involved. Several papers have derived such valuation formulas in a life-cycle framework under the assumption that people maximize expected utility (Usher, 1973; Conley, 1976; Arthur, 1981; Bergstrom, 1982; Jones-Lee and Poncelet, 1982; Shepard and Zeckhauser, 1984; Rosen, 1988; Ehrlich, 2000; Johansson, 2002). Empirical evidence abounds, however, that people often behave in ways that systematically violate expected utility (Camerer, 1995; Starmer, 2000). Even though cost–benefit analysis is a prescriptive activity, these descriptive violations are important, because the elicitation of willingness to pay is usually done descriptively and, hence, is affected by the violations of expected utility. Incorrectly assuming expected utility in analyzing responses to willingness to pay questions may lead to biased risk valuations and, consequently, to biased policy recommendations. There is a need to derive valuation formulas for changes in mortality risks that take into account that people deviate from expected utility.

An important reason why people deviate from expected utility is that they do not evaluate probabilities linearly, but distort probabilities. Many studies have demonstrated the importance of probability distortion in risky choice, both for monetary outcomes (Tversky and Kahneman, 1992; Camerer and Ho, 1994; Wu and Gonzalez, 1996; Gonzalez and Wu, 1999; Abdellaoui, 2000) and for health outcomes and life and death decisions (Bleichrodt and Pinto, 2000). There is also growing evidence that probability dis-

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tortion plays an important role in explaining a variety of field data (Camerer, 2000). For evidence of probability distortion in insurance decisions see, for example, Hershey and Schoemaker (1980) and Wakker et al. (1997, 2005). A formal theory of probability distortion is rank-dependent utility (Quiggin, 1981; Yaari, 1987).

This paper explores the impact of probability distortion on the evaluation of changes in risks to human life while paying attention to the effect of such distortions on the optimal intertemporal consumption choices. These choices depend not only on probability distortion but also on the changes in insurance opportunities offered by the changes in the probability of death. Our main result is that when people distort probabilities, the value of changes in survival risks is determined jointly by parameters that are already present under expected utility and by a new one: the elasticity of the probability weighting function. Using estimates from the empirical literature, we conclude that erroneously ignoring probability distortion will generally lead to cost–benefit ratios that are too high and that generate too much priority for activities that achieve risk reductions at younger ages.

The structure of the paper is as follows. In Section 2, we briefly explain rank-dependent utility. Sections 3 and 4 analyze the valuation of mortality risks under rank-dependent utility. To illustrate the main concepts involved, we start in Section 3 with the simple case in which there are just two periods. In Section 4, we extend the two-period model to a multi-period model. Section 5 discusses the interpretation of our findings and concludes the paper. Proofs and extensions are in Appendices A and B.

2. Rank-dependent utility

We consider an individual who has to make a choice between prospects. A typical *prospect* is $(p_1, x_1; \ldots; p_m, x_m)$, which gives monetary outcome x_i with probability p_i. Hence, prospects have finite support. The set of conceivable monetary amounts is a bounded positive interval $[0, M]$. The set of prospects P includes all *riskless prospects*, i.e. prospects that give one outcome with probability one. Riskless prospects are identified with the outcome they generate. The individual has preferences over P and, as usual, $\succcurlyeq$ denotes *weak preference*, $\succ$ *strict preference*, and $\sim$ *indifference*. Because P contains all riskless prospects, $\succcurlyeq$ also defines a preference relation over outcomes. It is implicit in the notation $(p_1, x_1; \ldots; p_m, x_m)$ that $x_1 \succcurlyeq \cdots \succcurlyeq x_m$, i.e. each prospect is *rank-ordered*. A function V *represents* $\succcurlyeq$ whenever for all prospects P and Q, $P \succcurlyeq Q$ iff $V(P) \geq V(Q)$. *Rank-dependent utility* holds if preferences over prospects can be represented by

$$\sum_{i=1}^{m} \pi_i U(x_i), \tag{1}$$

where U is a *utility function*, a real-valued function over outcomes. The utility function in rank-dependent utility is unique up to unit and location. The *decision weights* π_i are defined as

$$\pi_i = w\left(\sum_{j=1}^{i} p_j\right) - w\left(\sum_{j=1}^{i-1} p_j\right), \tag{2}$$

where w is a *probability weighting function*, it satisfies $w(0) = 0$ and $w(1) = 1$ and it is strictly increasing, i.e. if $p > q$ then $w(p) > w(q)$. *Expected utility* is the special case of rank-dependent utility where $w(p) = p$ for all p in $[0, 1]$. Eq. (2) shows that in rank-dependent utility the decision weights π_i are computed by taking the difference between the transformed probability of receiving at least x_i minus the transformed probability of receiving a better outcome than x_i.

Empirical and experimental studies have suggested that the probability weighting function is inverse S-shaped, overweighting low probabilities and underweighting high probabilities (Tversky and Kahneman, 1992; Gonzalez and Wu, 1999; Abdellaoui, 2000; Bleichrodt and Pinto, 2000; Camerer, 2000; Wakker et al., 2005). The point at which the function changes from overweighting probabilities to underweighting probabil-

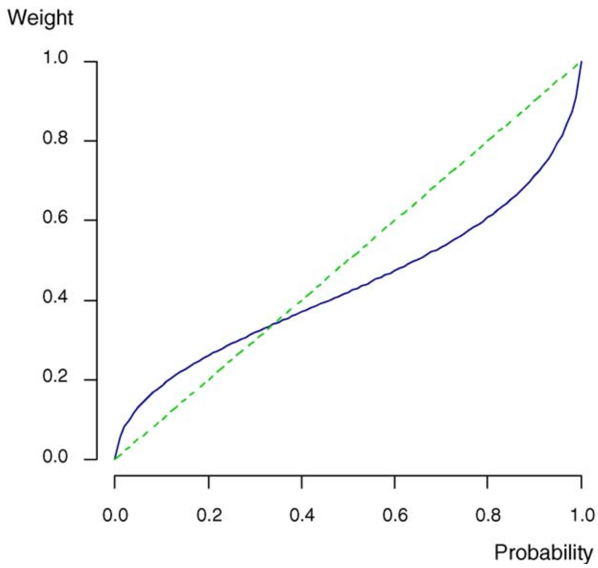

Fig. 1. An inverse S-shaped probability weighting function.

ities, i.e. the point at which $w(p) = p$, lies between 0.30 and 0.40. Fig. 1 displays such a probability weighting function. The dotted line shows the line where $w(p) = p$, the case in which expected utility holds. The probability weighting function is particularly steep, reflecting high sensitivity to changes in probability, for probabilities close to 0 and close to 1.

3. Two periods

3.1. Decision problem

Consider an individual with given initial wealth $W > 0$, who lives for at most two periods. This assumption is clearly unrealistic, but it will help to clarify the main concepts involved. The extension to more than two periods is presented in the next section. The individual lives for sure in the first period and has a probability p to survive to the second period. This probability is known to the individual. The individual's decision problem is to decide on consumption c_1 in the first period, with $0 \leq c_1 \leq W$. Hence, the individual cannot have debts. Consumption is monetary and utility is increasing and concave in consumption, i.e. $U'(c) > 0$ and $U''(c) < 0$.

We assume the stochastic equivalent of a perfect capital market in which actuarially fair life-assured annuities are available (Yaari, 1965). That is, the individual participates in an actuarial annuity system in which a cohort of identical individuals turn over their wealth to an insurance-finance company in exchange for a contract that guarantees consumption c_2 until death, where c_2 is the consumption in the second period, which will be specified below. The consumption risk of death is insured because those who die earlier than average effectively "subsidize" those who live longer than average. As Rosen (1988) argued, a model with actuarially fair annuities yields a value of changes in risks to life that is appropriate for cost–benefit analysis, because the costs of supporting the lives saved are deducted from the benefits derived from these lives. We assume that the insurance company does not distort probabilities, possibly because of its extensive experience with handling probabilities, and bases its actuarial calculations on expected value, because it has many customers and the law of large numbers applies to it. Then, the individual's budget constraint is equal to

$$W = c_1 + p \cdot \frac{c_2}{1 + r}, \tag{3}$$

where r is the market rate of interest. Because we assume that all individuals are identical, the same W and p apply to all individuals participating in the annuity system. Given the individual's budget constraint, his choice of c_1 immediately implies $c_2 = \frac{(W - c_1)(1+r)}{p}$.

We assume discounted utility with subjective discount rate a. That is, if the individual lives in the second period then his utility is $U(c_1) + \frac{U(c_2)}{1+a}$. So this is discounted utility with time separability as in Samuelson (1937). Throughout this section, we assume that the individual's subjective rate of discount equals the market rate of interest r, i.e. $a = r$. This assumption is made to clarify the effect of probability distortion on the evaluation of risks to life. We briefly consider relaxing this assumption in Appendix B. We assume that the individual has no heirs or altruism towards others and, hence, dying is identified with consuming $c_2 = 0$. We scale utility such that $U(0) = 0$. Therefore, the utility of a choice c_1 if the individual dies after period 1 is $U(c_1)$.

Given the above assumptions, the individual faces two options for a given choice of c_1: there is a probability p that his utility will be $U(c_1) + (U(c_2)/(1 + r))$ and a probability $1 - p$ that his utility will be $U(c_1)$. We assume that the individual maximizes rank-dependent utility. Because living during period 2 generates at least as much utility as being dead the individual's total utility is

$$\begin{aligned} \text{RDU} &= w(p) \cdot \left(U(c_1) + \frac{U(c_2)}{1 + r} \right) + (1 - w(p)) \cdot U(c_1) \\ &= U(c_1) + w(p) \cdot \frac{U(c_2)}{1 + r}. \end{aligned} \tag{4}$$

3.2. Optimal consumption

Theorem 1. *Suppose that the decision model of* Section 3.1 *holds. Then, for all p in* (0, 1):

(a) *if* $w(p) < p$ *then* $c_1 > c_2$;
(b) *if* $w(p) > p$ *then* $c_1 < c_2$;
(c) *if* $w(p) = p$ *then* $c_1 = c_2$.

We say that for all p in (0, 1) the individual is *pessimistic at p* if $w(p) < p$. The individual is *pessimistic* if $w(p) < p$ for all p in (0, 1), i.e. if he is pessimistic at all p in (0, 1). For all p in (0, 1), the individual is *optimistic at p* if $w(p) > p$. The individual is *optimistic* if $w(p) < p$ for all p in (0, 1). Theorem 1 shows that a pessimistic individual will always choose a decreasing consumption profile. Intuitively, this follows because the individual assigns a lower decision weight to being alive in period 2 than the insurance company does and, hence, effectively considers the insurance arrangement actuarially unfair. Theorem 1 also shows that an optimistic individual will always choose an increasing consumption profile and that an expected utility maximizer, who does not distort probabilities, will choose a constant consumption profile.

Compared with expected utility, pessimism leads to a reduction in future consumption and optimism leads to an increase in future consumption. The empirical findings on probability weighting, briefly reviewed in Section 2, therefore imply that if the survival probability p is less than (approximately) 0.35, the individual will choose an increasing consumption profile; else, he will choose a decreasing consumption profile.

3.3. Willingness to pay

We now determine the individual's willingness to pay for a change in mortality risk that affects all individuals involved in the insurance pool similarly. Summing individual willingness to pay over all individuals in the insurance pool will give societal willingness to pay for this increase in the survival probability.

Theorem 2. *Suppose that the decision model of* Section 3.1 *holds. Then, the individual willingness to pay for a change in mortality risk that affects all individuals in the insurance pool in the same way is equal to* $\frac{c_2}{1+r}\frac{\varphi_p-\varepsilon_{c_2}}{\varepsilon_{c_2}}$, *where* $\varphi_p = p \cdot \frac{w'(p)}{w(p)}$ *denotes the elasticity of the probability weighting function at p and* $\varepsilon_{c_2} = c_2 \cdot \frac{U'(c_2)}{U(c_2)}$ *denotes the elasticity of the utility function at* c_2.

Theorem 2 says that the willingness to pay for reductions in mortality risk is positively associated with consumption in period 2 and with the elasticity of the probability weighting function and negatively with the interest rate and the elasticity of the utility function. The parameter ε_{c_2} reflects both the possibilities for intertemporal substitution of consumption and the value of being alive relative to being dead and is discussed in detail by Rosen (1988).

Theorem 2 shows that the individual is willing to pay for reductions in mortality risk if and only if $\varepsilon_{c_2} < \varphi_p$. If $\varepsilon_{c_2} > \varphi_p$ then the individual is willing to pay for increases in mortality risk. This possibility may appear counterintuitive, but can arise because an increase in the survival probability has two effects; see Fig. 2 for an illustration. First, an increase in the survival probability increases rank-dependent utility $U(c_1) + w(p) \cdot \frac{U(c_2)}{1+r}$. This effect is illustrated in Fig. 2A. The initial optimal point is point A and the slope of the indifference curve there is $\frac{U'(c_1)}{w(p)U'(c_2)}$, assuming for ease of illustration that the subjective rate of discount is zero. Suppose now that the probability of survival increases to p^*. The new indifference curve, the curve RS in Fig. 2A, is shallower than the original indifference curve because its slope is equal to $\frac{U'(c_1)}{w(p^*)U'(c_2)}$ and $w(p^*) > w(p)$ by the probability weighting function being strictly increasing. The difference in slope depends on the elasticity of the probability weighting function. The level of utility at A is now $U(c_1^*) + w(p^*)U(c_2^*)$, which is larger than the utility obtained at the original indifference curve. Besides, A is no longer optimal at the original budget line and, hence, the individual can further increase his utility. If the budget line were not affected there would be an increase in utility caused by the increase in the survival probability and the size of this increase depends on the elasticity of the probability weighting function.

The budget line is, however, affected; this second effect of the increase in the survival probability is illustrated in Fig. 2B. Due to the increase in survival probability, more individuals in the insurance pool survive and the total wealth has to be shared between more sur-

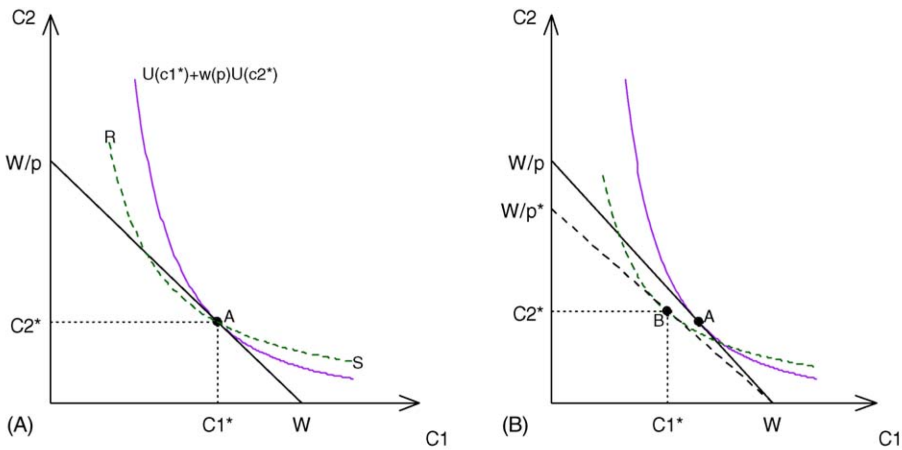

Fig. 2. (A and B) The influence of a change in the survival probability.

vivors. As a result, the budget line shifts to the new line $W - W/p^*$. That is, the individual's consumption opportunities are reduced. The extent to which this reduction in consumption opportunities affects utility depends on the elasticity of the utility function. The more elastic the utility function is, the larger is the decrease in utility.

The combined effect is shown in Fig. 2B by the shift from point A to point B. Whether the individual is willing to pay for the increase in survival probability depends on whether the utility at B exceeds the utility at A. If the elasticity of the utility function is high relative to the elasticity of the probability weighting function then willingness to pay can be negative. In that case the positive effect of the increase in p is relatively small, because the individual is insensitive to changes in p, compared with the negative effect due to the reduced consumption opportunities and the resulting decrease in the utility of consumption. In the extreme case where the elasticity of the probability weighting function is zero, the new and the original indifference curves coincide and the increase in the survival probability clearly reduces the individual's utility.

3.4. Illustrations

We noted above that c_2 depends among other things on whether the individual is pessimistic or optimistic. For the inverse S-shaped probability weighting function this means that c_2 will tend to decrease with the survival probability. However, φ_p tends to increase with p given inverse S-shaped probability weighting. For example, if we take Prelec's (1998) weighting function

$$w(p) = \mathrm{e}^{-\alpha(-\ln p)^{\beta}}, \tag{5}$$

which must have $\alpha > 0$, $0 < \beta < 1$, to satisfy the requirements of an inverse S-shaped probability weighting function, then

$$\varphi_p = \alpha\beta(-\ln p)^{\beta-1}, \tag{6}$$

and φ_p is increasing in probability. Hence, Theorem 2 shows that the effect of survival probability on the willingness to pay for a change in mortality risk is sign-ambiguous.

In the expression for the willingness to pay (Theorem 2), for small probabilities the effect of probability weighting on c_2 will generally outweigh the effect of φ_p, but for large probabilities the effect of φ_p will dominate. Ceteris paribus, this suggests a U-shaped relationship between willingness to pay and survival probability. In particular, for high survival probabilities willingness to pay will increase with the survival probability. This prediction is consistent with the available empirical evidence (Smith and Desvousges, 1987). As was shown by Smith and Desvousges (1987), such an increasing relationship between survival probability and willingness to pay cannot be explained under expected utility.

A simple numerical example may illustrate. Suppose ε_{c_2} is constant and equal to 0.25, a value which is consistent with the data in Thaler and Rosen (1975). Suppose further that the probability weighting function is equal to Prelec's function (Eq. (5)) with the parameter estimates $\alpha = 1.08$ and $\beta = 0.53$ obtained by Bleichrodt and Pinto (2000) and that $r = 0.10$. Fig. 3A shows that c_2 is generally decreasing with the survival probability and this effect is particularly strong for low survival probabilities. There is only a small increase in c_2 for high survival probabilities.[1] The elasticity φ_p rises, however, with the survival probability, as is shown in Fig. 3B, and this effect is particularly strong for high survival probabilities. Fig. 3C shows that willingness to pay indeed has the predicted U-shape: it decreases up to $p = 0.28$ and increases from $p = 0.28$ onwards.

3.5. A comparison between rank-dependent utility and expected utility

3.5.1. Same degree of utility curvature

We will assume that ε_c is constant. This assumption was also made in Thaler and Rosen (1975) and corresponds to utility being a power function. The power utility function is often used in economics and decision analysis. Under expected utility, $\varphi_p = 1$ for all p in [0, 1]. Assume for the time being that the utility function in rank-dependent utility is equal to the utility function in expected utility. That is, we effectively study the pure impact of the introduction of probability distortion on the willingness to pay for changes in mortality risks. Theorem 2 shows that this impact depends on whether φ_p is larger or smaller than 1 and on the effect of probability distortion on c_2. We observed in Theorem 1 that if the individual is optimistic (pessimistic) then probability distortion will lead to an increase (decrease) in c_2 in comparison with expected utility. Hence, we obtain:

Corollary 3. *Suppose that the decision model of* Section 3.1 *holds, that the utility function in rank-dependent utility is identical to the utility function in expected utility, and that ε_c is constant. Then, for all p in* (0, 1), *if*

[1] Bleichrodt and Pinto (2000) did not actually measure probability weighting for extreme probabilities so the observations for such extreme probabilities should be interpreted with care.

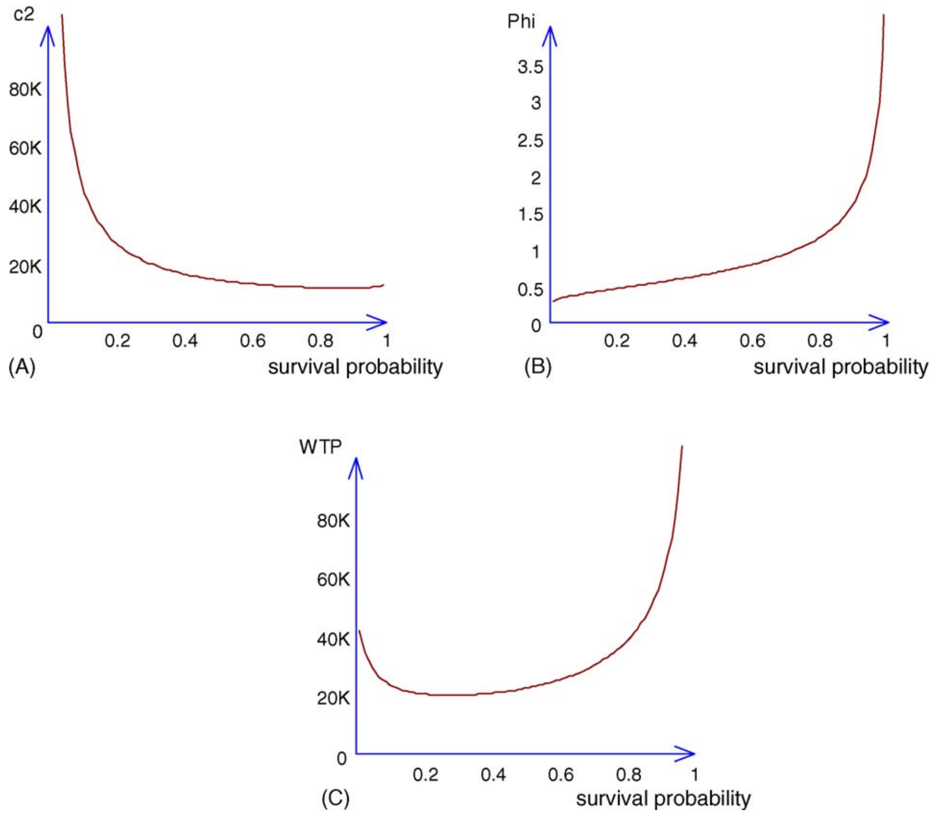

Fig. 3. The influence of survival probability on c_2, φ_p, and WTP: (A) the relationship between c_2 and p; (B) the relationship between φ and p; (C) the relationship between WTP and p.

$\varphi_p > 1$ *and the individual is optimistic at p, probability distortion leads to an increase in willingness to pay for changes in mortality risk. If* $\varphi_p < 1$ *and the individual is pessimistic at p, probability distortion leads to a decrease in willingness to pay for changes in mortality risk.*

In the other cases (e.g. if $\varphi_p > 1$ and the individual is pessimistic at p or if $\varphi_p < 1$ and the individual is optimistic at p) it is not possible to make statements about the effect of probability distortion on willingness to pay for changes in mortality risk that are generally valid. Suppose that Prelec's (1998) probability weighting function holds with the estimates obtained by Bleichrodt and Pinto (2000). Then, $w(p) \leq p$ for p exceeding 0.30 and $\varphi_p \leq 1$ for p less than 0.74. Hence, probability distortion will decrease the willingness to pay for reductions in mortality risk if the survival probability lies between 0.30 and 0.74. For other values of p the effect is sign-ambiguous.

Fig. 4A shows the effect of the introduction of probability distortion using the numerical example of the previous subsection. The figure shows the ratio of willingness to pay after the introduction of probability distortion and willingness to pay under expected utility. The dotted horizontal line serves as a benchmark and corresponds to no impact of probability distortion. The introduction of probability distortion leads to a fall in willingness to pay when the survival probability is less than 0.81 and to an increase in willingness to pay when the survival probability exceeds 0.81. The difference can be large. If p is around 0.20, the degree of optimism and, hence, the difference between the optimal c_2 under rank-dependent utility and under expected utility is small, but φ_p is also small leading to a willingness to pay that is nearly three times lower than under expected utility. If $p = 0.99$ then the degree of pessimism is small, but φ_p is high and the willingness to pay is almost six times higher than under expected utility.

3.5.2. Different degrees of utility curvature

Let us now drop the assumption that the utility function in rank-dependent utility is equal to the utility function in expected utility. Several authors have shown that

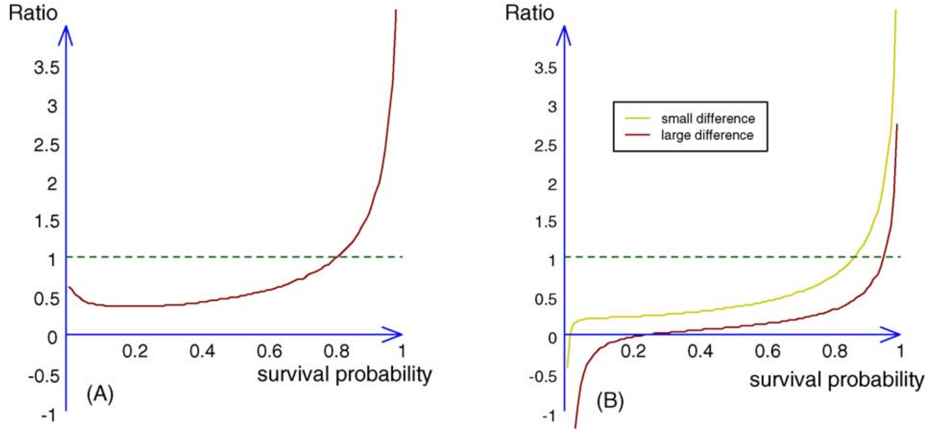

Fig. 4. The influence of probability distortion on WTP: (A) same utility and (B) different utility.

under rank-dependent utility, the utility function is less concave, hence closer to linearity, than under expected utility. See Edwards (1955), Fox et al. (1996), Selten et al. (1999), Luce (2000), Rabin (2000), and Diecidue and Wakker (2002) when the outcome domain consists of moderate amounts of money and Wakker and Deneffe (1996) and Bleichrodt et al. (1999) when the outcome domain consists of life durations. This finding can be explained by the fact that under expected utility all risk attitude is captured in the utility function, whereas under rank-dependent utility part of the risk attitude is modeled through the probability weighting function.

In general, we cannot conclude what will happen to the elasticity of a utility function when it becomes less concave. However, if utility is a power function then the elasticity will increase when the utility function becomes less concave and the cost of the increase in the survival probability, decreased consumption opportunities because more individuals in the pool survive, will get more weight. Hence, willingness to pay will fall ceteris paribus.

The difference in curvature of utility also affects the difference between the optimal values of c_2 under rank-dependent utility ($c_{2_{\mathrm{RDU}}}$) and under expected utility ($c_{2_{\mathrm{EU}}}$). If a utility function becomes less concave then its marginal utility will decrease at a lower rate. Consequently, $|c_{2_{\mathrm{RDU}}} - c_{2_{\mathrm{EU}}}|$ will increase in comparison with the case where we assumed identical utility functions leading, ceteris paribus, to a lower willingness to pay when $w(p) < p$ and to a higher willingness to pay when $w(p) > p$. Hence, we obtain:

Corollary 4. *Suppose that the decision model of* Section 3.1 *holds. Suppose also that the utility function in rank-dependent utility is less concave than the utility function in expected utility. If utility is a power function then for all p in* (0, 1) *the willingness to pay for changes in mortality risk under rank-dependent utility will fall compared with the situation where we assumed that the utility function in rank-dependent utility was the same as in expected utility when the individual is pessimistic at p.*

Suppose that the utility function is a power function. Then, under the empirically observed inverse S-shaped probability weighting function, the effect of less concave utility under rank-dependent utility than under expected utility will be a decrease in willingness to pay under rank-dependent utility. This is clearly the case if $p > 0.35$ because then c_2 falls and ε_{c_2} rises. For $p < 0.35$, φ_p is close to ε_{c_2}, and the effect of a change in ε_{c_2} on the ratio $\frac{\varphi_p - \varepsilon_{c_2}}{\varepsilon_{c_2}}$ will be much larger than its effect on c_2. Consequently, willingness to pay will fall.

Fig. 4B shows the effect of different utility in the numerical example described afore. We considered two deviations in utility: "small", $\varepsilon_{c_2} = 0.30$ under rank-dependent utility and $\varepsilon_{c_2} = 0.25$ under expected utility, and "large", $\varepsilon_{c_2} = 0.50$ under rank-dependent utility and $\varepsilon_{c_2} = 0.25$ under expected utility, leaving all other parameters unchanged. The pattern is similar in both cases, although obviously more pronounced for the larger deviation: willingness to pay under rank-dependent utility falls compared with expected utility. In fact, willingness to pay becomes negative under rank-dependent utility for small probabilities for the small deviation and up till $p = 0.28$ for the large deviation. Only for high survival probabilities is the willingness to pay higher under rank-dependent utility than under expected utility (for p higher than 0.87 for the small deviation and for p higher than 0.96 for the large deviation).

4. More than two periods

4.1. Decision model

We now consider the case where there are more than two periods. The decision model is the same as that in Section 3.1 except for the following. The individual lives a maximum of T periods where T is an upper bound on the length of life. Let p_t be the probability that the individual survives during period t, $t \in \{1, \ldots, T\}$. Then, the probability of being alive at period t, S_t, is equal to $S_t = \prod_{\tau=1}^{t-1} p_\tau$. The S_t are known to the individual. As in Section 3.1, $S_1 = 1$, i.e. the individual lives for sure in the first period. We assume that $S_t \neq 0$ for all $t \in \{1, \ldots, T\}$. The individual's decision problem is to decide on the c_t's, $t = 1, \ldots, T-1$, where for all $t \in \{1, \ldots, T-1\}$, $0 \leq \sum_{j=1}^{t} c_j \leq W$. Again, we assume that the individual's subjective rate of discount equals the rate of time preference and we assume that the market rate of interest is constant over time and equal to r. In Appendix B, we consider the effects of relaxing these assumptions. As before, the individual participates in the Yaari-type social insurance arrangement.

The decision weights of a rank-dependent utility maximizer are computed by transforming the S_t. Because living longer is always preferred under the Yaari-type social insurance arrangement, for any consumption profile the best outcome is living for T periods in which case the individual enjoys utility

$$\sum_{t=1}^{T} \frac{U(c_t)}{(1+r)^{t-1}}, \tag{7}$$

which obtains with probability S_T. Hence, the decision weight is $w(S_T)$. In general, the decision weight of living exactly τ periods and enjoying utility $\sum_{t=1}^{\tau} \frac{U(c_t)}{(1+r)^{t-1}}$ is $w(S_\tau) - w(S_{\tau+1})$. It follows from some simple algebraic manipulation that the individual's rank-dependent utility is equal to

$$\mathrm{RDU} = \sum_{t=1}^{T} w(S_t) \frac{U(c_t)}{(1+r)^{t-1}}, \tag{8}$$

and his budget constraint is equal to

$$W = \sum_{t=1}^{T} S_t \frac{c_t}{(1+r)^{t-1}}. \tag{9}$$

4.2. Optimal consumption

Theorem 5. *Suppose that the decision model of* Section 4.1 *holds. Then, for all t, consumption in period t increases with the ratio* $\frac{w(S_t)}{S_t}$.

Theorem 5 resembles Theorem 1: consumption in period t is higher the more weight is given to the probability of being alive in period t. If the probability weighting function is inverse S-shaped then the relationship between $\frac{w(S_t)}{S_t}$ and S_t and, hence, by Theorem 5, the relationship between c_t and S_t, is inverse U-shaped. Consumption will be relatively high in those years in which the probability of being alive is relatively low. It also follows from Theorem 5 that consumption is constant over time under expected utility.

4.3. Willingness to pay

We determine the individual's willingness to pay for an age-specific change in age-specific mortality risk so that the survival function S_t becomes $S_t + \delta S_t$ with $\delta S_t > 0$ and $S_t + \delta S_t \leq 1$. Hence, we analyze the effect of an increase in p_τ for one given τ. By definition, this affects not only S_τ but also S_t for all $t > \tau$. We assume that the change in age-specific risks affects all individuals in the insurance pool in the same way.

Theorem 6. *Suppose that the decision model of* Section 4.1 *holds. Then, the individual's willingness to pay for an age-specific change in age-specific mortality risk that affects all individuals in the insurance pool in the same way is equal to* $\sum_{t=1}^{T} \left(\frac{c_t}{(1+r)^{t-1}} \left(\frac{\varphi_{S_t} - \varepsilon_{c_t}}{\varepsilon_{c_t}}\right) \delta S_t\right)$, *where* φ_{S_t} *denotes the elasticity of the probability weighting function at survival probability* S_t *and* ε_{c_t} *denotes the elasticity of the utility function at consumption* c_t.

Theorem 6 is similar to Theorem 2. The willingness to pay for changes in mortality risk increases with consumption and with the elasticity of the probability weighting function and decreases with the interest rate and with the elasticity of the utility function. As in Section 3, if the probability weighting function has an inverse S-shape then φ_{S_t} will generally increase in S_t. On the other hand, if the probability weighting function is inverse S-shaped then c_t will be a U-shaped function of S_t. Therefore, the effect of S_t on willingness to pay is sign-ambiguous. However, if the probability weighting function is inverse S-shaped and the utility function does not diverge "too much" from linearity, i.e. ε_{c_t} does not differ "too much" from 1, then the effect of φ_{S_t} will generally dominate and, consequently, willingness to pay will generally be higher for changes in S_t that occur in years with higher survival probability.

4.4. A comparison between rank-dependent utility and expected utility

We now compare willingness to pay under rank-dependent utility and under expected utility. We start, as in Section 3, with the case where the utility function in rank-dependent utility is the same as the utility function in expected utility. That is, we study the "pure effect" of the introduction of probability distortion on willingness to pay. We then drop this assumption and study the empirically realistic case where the utility function in rank-dependent utility is less concave than the utility function in expected utility. We will show that the conclusions of Corollaries 3 and 4 remain valid in the multi-period setting.

4.4.1. Same degree of utility curvature

We assume that ε_{c_t} is constant. Under expected utility, $\varphi_{S_t} = 1$ for all S_t in [0, 1] and, by Theorem 5, consumption is constant. From Theorem 6, it follows that under expected utility the willingness to pay for changes in the mortality schedule is equal to:

$$\sum_{t=1}^{T}\left(\frac{c}{(1+r)^{t-1}}\left(\frac{1-\varepsilon_{c_t}}{\varepsilon_{c_t}}\right)\delta S_t\right). \tag{10}$$

A comparison between Eq. (10) and Theorem 6 reveals that the effect of probability distortion depends on the sign of $c_t - c$ and on φ_{S_t}. It leads to a higher willingness to pay if $c_t > c$ and $\varphi_{S_t} > 1$ and to a lower willingness to pay if $c_t < c$ and $\varphi_{S_t} < 1$. By Theorem 5, c_t will exceed c when the individual is optimistic at S_t and c_t will fall short of c when the individual is pessimistic at c_t. Hence, Corollary 3 still holds in the multi-period setting.

Assume, for illustration, that probability distortion can be described by Prelec's (1998) weighting function (Eq. (5)) with the parameter estimates from Bleichrodt and Pinto (2000). Then, we find, as in Section 3.5, that years in which the survival probability lies between 0.30 and 0.74 contribute to a higher willingness to pay under expected utility. For the other years, the effect is sign-ambiguous. If the survival probability is high then the effect of the elasticity of the probability weighting function, φ_{S_t}, will dominate the effect of pessimism on consumption. Hence, for years in which the survival probability is high, willingness to pay will be higher under rank-dependent utility. That is, probability distortion leads to more favorable willingness to pay estimates for activities that save the young.

4.4.2. Different degrees of utility curvature

Relaxing the assumption that the utility function in rank-dependent utility is equal to the utility function in expected utility affects both ε_{c_t} and the c_t. As in the two-period case, we cannot draw definite conclusions without imposing additional assumptions on the utility function. Suppose that utility is a power function and, hence, that its elasticity is constant. Assume further the empirically plausible case where the utility function in rank-dependent utility is less concave, i.e. closer to linearity, than the expected utility function. Then, ε_{c_t} will be larger under rank-dependent utility than under expected utility, leading to a lower willingness to pay under rank-dependent utility, ceteris paribus. Also, $|c_t - c|$ will rise, leading to a lower willingness to pay in years in which $w(S_t) < S_t$ and to a higher willingness to pay in years in which $w(S_t) > S_t$, ceteris paribus. Hence, Corollary 4 still holds.

If utility is a power function, then, as in the two-period case, willingness to pay will fall under rank-dependent utility when allowance is made for less concave utility and probability weighting is described by the inverse S-shaped probability weighting function most commonly found in the literature. If $S_t > 0.35$ this follows from Corollary 4. For $S_t < 0.35$, φ_{S_t} is close to ε_{c_t}, and the effect of a change in ε_{c_t} on the ratio $\frac{\varphi_{S_t}-\varepsilon_{c_t}}{\varepsilon_{c_t}}$ will be much larger than its effect on c_t. Consequently, willingness to pay will fall. For example, for Prelec's (1998) probability weighting function with the estimates obtained by Bleichrodt and Pinto (2000), φ_{S_t} never exceeds 0.52 in the range where probabilities are overweighted. Hence, a modest increase in ε_{c_t} from 0.25 to 0.30 will lead to a decrease in $\frac{\varphi_{S_t}-\varepsilon_{c_t}}{\varepsilon_{c_t}}$ of at least 30%, whereas the effect on c_t is small.

5. Conclusions

This paper shows that if people distort probabilities then a cost–benefit analysis that looks at people's responses as if they behave according to expected utility will generally use willingness to pay estimates that are too low. The exception is when programs are considered that lead to risk reductions at younger ages. Hence, our main conclusion is that erroneously ignoring probability distortion in cost–benefit analysis will generally lead to cost–benefit ratios that are too high and that generate too much priority for programs that save "young lives". As we showed by numerical examples in Section 3, the bias due to probability distortion can be large, in particular in the plausible case where the utility function in rank-dependent utility is less concave than the utility function in expected utility.

For conceptual clarity, we made several simplifying assumptions. In Appendix B, we show that dropping the assumptions that the individual's subjective rate of discount is equal to the market rate of interest and that discounting is constant does not affect our main results, Theorems 2 and 6. The model could of course be further extended to encompass, for example, intentional bequest motives (Chang, 1991), altruism, and the inclusion of quality of life as an argument in the utility function. Such extensions will undoubtedly make the definition of the value of changes in risks to human life more complicated. However, it is unlikely that they will affect the general message of this paper regarding the effect of probability distortion on cost–benefit ratios.

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Appendix A. Proofs

Proof of Theorem 1. The individual's decision problem is

$$\underset{c_1}{\operatorname{Max}} L = U(c_1) + w(p)\cdot\frac{1}{1+r}U\left(\frac{(W-c_1)(1+r)}{p}\right) \tag{A1}$$

and the first-order condition is

$$\frac{\partial L}{\partial c_1} = U'(c_1) - \frac{w(p)}{p}U'\left(\frac{(W-c_1)(1+r)}{p}\right) = 0. \tag{A2}$$

The second-order condition is

$$\frac{\partial^2 L}{\partial c_1^2} = U''(c_1) - \frac{w(p)}{p^2}(1+r)U''\left(\frac{(W-c_1)(1+r)}{p}\right), \tag{A3}$$

which is negative by the concavity of U. Hence, we obtain an interior solution.

Because $c_2 = \frac{(W-c_1)(1+r)}{p}$, the first-order condition shows that for a given p, $U'(c_2) \gtreqless U'(c_1)$ if and only if $w(p) \lesseqgtr p$. Because utility is increasing and concave in consumption, $w(p) \lesseqgtr p$ implies $c_2 \lesseqgtr c_1$. $\square$

Proof of Theorem 2. By applying the envelope theorem to (A1), the willingness to pay for a change in the mortality probability p is equal to

$$\frac{\mathrm{d}W}{\mathrm{d}p} = \frac{1}{1+r}\left(\frac{pw'(p)}{w(p)}\frac{U(c_2)}{U'(c_2)} - c_2\right) = \frac{c_2}{1+r}\frac{\varphi_p - \varepsilon_{c_2}}{\varepsilon_{c_2}}. \quad \square$$

Proof of Theorem 5. The individual's decision problem is

$$\underset{c_1,\ldots,c_{T-1}}{\operatorname{Max}} = \sum_{t=1}^{T} w(S_t)\frac{U(c_t)}{(1+r)^{t-1}} \tag{A4a}$$

with

$$W = \sum_{t=1}^{T} S_t \frac{c_t}{(1+r)^{t-1}}. \tag{A4b}$$

The budget constraint (A4b) can be rewritten as

$$c_T = \frac{(1+r)^{T-1}}{S_T}\left(W - \sum_{t=1}^{T-1} S_t\frac{c_t}{(1+r)^{t-1}}\right). \tag{A5}$$

Introducing (A5) into (A4a) and differentiating with respect to $c_1, \ldots, c_{T-1}$, we obtain after some algebraic manipulations, the optimality conditions

$$\frac{U'(c_t)}{U'(c_{t'})} = \frac{\frac{w(S_{t'})}{S_{t'}}}{\frac{w(S_t)}{S_t}}, \quad \text{for any } t, t' \text{ in } \{1, \ldots, T-1\}. \tag{A6}$$

The second-order condition for a maximum is satisfied, because utility is additive over time and because of the concavity of the utility function and the linearity of the budget constraint in each c_t.

From (A6) and the concavity of U, we obtain that $c_t \gtreqless c_{t'}$ if and only if $\frac{w(S_t)}{S_t} \gtreqless \frac{w(S_{t'})}{S_{t'}}$. Hence, consumption will increase with the ratio $\frac{w(S_t)}{S_t}$. $\square$

Proof of Theorem 6. Because rank-dependent utility depends on the entire T-tuple S_t we must study the effect of variations in the T-tuple S_t. This is technically a Fréchet derivative, i.e. a multi-dimensional derivative. Let us use the symbol δ to denote this type of differentiation. From (A4a) and (A5) we obtain

$$0 = \sum_{t=1}^{T}\left((w'(S_t)\frac{U(c_t)}{(1+r)^{t-1}} - \frac{w(S_T)}{S_T}U'(c_T)\frac{c_t}{(1+r)^{t-1}}\right)\delta S_t + \left(\frac{w(S_T)}{S_T}U'(c_T)\right)\mathrm{d}W, \tag{A7}$$

which yields

$$\delta W[\delta S_t] = \frac{\sum_{t=1}^{T}\left(w'(S_t)\frac{U(c_t)}{(1+r)^{t-1}} - \frac{w(S_T)}{S_T}U'(c_T)\frac{c_t}{(1+r)^{t-1}}\right)\delta S_t}{\frac{w(S_T)}{S_T}U'(c_T)} = \sum_{t=1}^{T}\left(\frac{c_t}{(1+r)^{t-1}}\left(\frac{\varphi_{S_t} - \varepsilon_{c_t}}{\varepsilon_{c_t}}\right)\delta S_t\right), \quad \text{(A8)}$$

where we used the optimality conditions

$$U'(c_t) = \frac{w(S_T)}{S_T}\frac{S_t}{w(S_t)}U'(c_T) \quad \text{(A9)}$$

in the second step and where $\delta W[\delta S_t]$ denotes the change in wealth due to the change in the survival probabilities.[2] □

Appendix B. Difference between the subjective rate of discount and the market rate of interest

We will now briefly show that dropping the assumptions that the individual's subjective rate of discount equals the market rate of interest and that discounting is constant does not change our main conclusions, Theorems 2 and 6. If $a \neq r$ Theorem 1 becomes:

Theorem A1. *Suppose that the decision model of* Section 3.1 *holds except that* $a \neq r$. *Then, for all p in* (0, 1), *consumption in period t increases with the ratio* $\frac{w(p)}{p}\frac{1+r}{1+a}$.

The proof follows by substituting a for r as the subjective rate of discount in (A1) and (A2). Theorem A1 shows that if the individual's subjective rate of discount exceeds the market rate of interest, which is often observed in empirical studies on time preference (Frederick et al., 2002) then c_2 will fall compared to the situation where we assumed that $a = r$.

It is easily verified that Theorem 2 is not affected by dropping the assumption that $a = r$. The decrease in c_2 resulting if $a > r$ will thus lead to a reduction in the willingness to pay for increases in the probability of survival compared with the situation where we assumed that $a = r$.

Because Theorem 2 still holds, Corollary 3 also holds. As regards Corollary 4, if the power utility function in rank-dependent utility is less concave than the power utility function in expected utility then, compared with the case $a = r$, the drop in c_2, and hence of willingness to pay, which occurs because $a > r$ will be larger under rank-dependent utility than under expected utility. This follows because marginal utility then falls at a slower rate under rank-dependent utility than under expected utility and, consequently, the decrease in c_2 that is required to ensure that the first-order conditions are fulfilled has to be larger under rank-dependent utility.

In the multi-period model we will not only assume that the subjective rate of discount and the market rate of interest differ, but we will also drop the assumption that the subjective rate of discount and the market rate of interest are constant over time. Violations of constant rate discounting are commonly observed in the empirical literature. Let a_t denote the individual's subjective rate of discount at time point t and let r_t denote the market rate of interest at time point t with $a_1 = r_1 = 0$. Theorem 5 then becomes:

Theorem A5. *Suppose that the decision model of* Section 4.1 *holds except that a and r may vary over time and for all t,* a_t *and* r_t *may be distinct. Then, for all t, consumption in period t increases with the ratio* $\frac{w(S_t)}{S_t}\prod\frac{1+r_t}{1+a_t}$.

The proof follows from appropriate substitution in (A4)–(A6). Empirical studies show that a_t tends to fall with time leading, ceteris paribus, to an increasing consumption profile over time. It is easily verified that Theorem 6 is not affected by allowing for non-constant discounting and dropping the assumption that for all t, $a_t = r_t$. Hence, our conclusions about the "pure effect" of probability distortion on the valuation of survival risks are not affected either. If power utility in rank-dependent utility is less concave than power utility in expected utility then, in comparison with the situation where $a = a_t = r_t = r$, both the decrease in c_t, and hence of willingness to pay, which occurs when $a_t > r_t$ and the increase in c_t and, hence, of willingness to pay, which occurs when $a_t < r_t$, will be larger under rank-dependent utility than under expected utility.

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[2] This notation is adopted from Arthur (1981).

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